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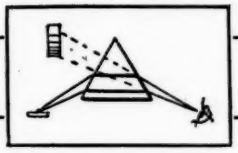

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Journal of the
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The Relationship Between Reciprocity Law Failure and the Intermittency Effect in Photographic Exposure

J. H. WEBB, *Eastman Kodak Research Laboratories**

(Received March 4, 1933)

INTRODUCTION

THE purpose of this experimental work was to investigate the relationship which exists between the two phenomena, reciprocity law failure and the intermittency effect, encountered in photographic exposure. Recent measurements by various experimenters[1] have indicated, at least for low values of intensity, that there is a close connection between these two effects. The results obtained by the present author have confirmed these results of other workers and have extended them to a wider range of intensity values than heretofore covered. A theoretical consideration of the results here obtained has led to a means for estimating the area on a photographic grain effective as the receptor of light quanta in exposure.

For the sake of clearness, the following explanation of the terms reciprocity law failure and intermittency effect as used in this paper are given:

If density D, the negative logarithm of transmission, be expressed as a function of the single variable U, $D=f(U)$, where U is defined as insolation, then the reciprocity law failure is concerned with the relationship which exists between U and the variables I (intensity) and t (time) of an exposure. A relationship between

* Communication No. 500.

[1] E. A. Baker, "On the Validity of Talbot's Law for the Photographic Plate," Proc. Opt. Soc. Convention, 238, Part I, 1926.

A. E. Weber, "Über die Anwendung des rotierenden Sektors zur photographischen Photometrie," Ann. d. Physik **45**, 801 (1914).

F. Eckert and R. Pummerer, "Photographische Spektralphotometrie der Absorpionspektren von Farbstoffen," Zeits. f. physik. Chemie **87**, 605 (1914).

K. S. Gibson, H. J. McNicholas, E. P. T. Tyndall, M. K. Frehafer and W. E. Mathewson, "The Spectral Transmissive Properties of Dyes," Bur. Stand. Sci. Pap. **18**, No. 440, 121 (1923).

F. Twyman and F. Simeon, "The Logarithmic Wedge Sector and Its Use in Quantitative Spectrum Analysis," Trans. Opt. Soc. (London) **31**, 169 (1929–30).

F. Twyman and A. Harvey, "The Validity of the Schwarzschild Relation as Applied to the Use of the Logarithmic Sector," Trans. Opt. Soc. (London) **33**, 1 (1931–32).

H. E. Howe, "On a Modification of the Hilger Sector Photometer Method for Measuring Ultra-Violet Absorption and Its Application in the Case of Certain Derivatives of Fluoran," Phys. Rev. **8**, 674 (1916).

B. O'Brien, "Photographic Spectral Energy Measurement with a Spiral Aperture Disc," Phys. Rev. **33**, 640 (1929).

B. O'Brien and E. Dickerman, "Intermittent Exposure in Photographic Spectrophotometry Over Wide Intensity Ranges," Phys. Rev. **37**, 471 (1931).

B. O'Brien and V. L. Parks, "Photographic Reciprocity and Intermittency Defects Near the Long Wave-length Limit of Plate Sensitivity," Phys. Rev. **41**, 387 (1932).

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these quantities may be expressed symbolically as $U=\phi(I, t)$. The Bunsen Roscoe reciprocity law gives to the function ϕ, the form, $\phi(I, t)=Ixt$. According to this law, equal values of energy, It, falling upon a photographic emulsion will produce equal densities regardless of the absolute level of intensity at which the exposures are made. In general, photographic materials do not obey this law and failure to do so is termed failure of the photographic reciprocity law.

Relative to the intermittency effect, it may be stated that a continuous exposure of energy It does not in general produce the same photographic density as a second, equal energy exposure given in a number of discreet installments. Failure of a photographic material to thus integrate correctly an exposure given in this latter manner is known as the intermittency effect.

Apparatus and Experimental Procedure

The exposures were made on the reciprocity sensitometer which has been used in all the previous experimental work[2] on reciprocity failure carried on in this laboratory. For a complete description of this sensitometer, the reader is referred to a paper on this instrument.[3] Since this paper was written the reciprocity sensitometer and its auxiliary apparatus have been dismantled and reinstalled in a new location, in a specially designed room for this work. The new installation of this instrument is slightly different from the old. In the new set-up, the instrument has been turned through 180 degrees with respect to the direction of the lamp track, the sector wheel and sector wheel cover having been reversed, so that now exposures are made on the opposite side of the apparatus from that in the former arrangement. This change was made for convenience and economy of space, and should have no effect upon the experimental results obtained. The sensitometer itself has a logarithmic sector wheel possessing ten power of two steps for varying the times of exposure. Characteristic (density *versus* log It) curves were always taken during a single revolution of this sector wheel. The time of revolution of the sector could be varied in steps, by means of a system of gear changes, from 0.0625 second to 72 hours, 49 minutes, 4 seconds. A four by five inch plateholder, placed directly behind the openings in the sector wheel, held the film or plate for exposure.

Extending out from the exposure plane of the sensitometer is a lamp track twelve meters in length upon which the light source used in making the exposures was placed. The light intensity was varied by moving the source along this track, the value of the intensity falling off inversely as the square of the distance. In order to vary the light intensity over a wider range of values than that permitted by the length of track, it was possible to reduce the source intensity by diaphragming the ground glass screen placed over the window of the lamp house. By so diaphragming the source, the length of track was again rendered available for further variation of the intensity. In the new installation there was no covering for the lamp track and it was necessary to darken the entire room during exposures. By the use of a number of dull black diaphragms placed at suitable intervals along the lamp track and a short black hood placed at the exposure plane, the small amount of stray light which was present in the room from the exposing lamp was sufficiently well screened out to eliminate errors from this source. This was tested by leaving a piece of high-speed negative film open in the sensitometer, with the direct exposing light cut off, for a period exceeding that of any of the experimental exposures. Upon development, no fogging was noted on this film.

---

[2] L. A. Jones and E. Huse, "On the Relation between Time and Intensity in Photographic Exposure," J. Opt. Soc. Am. and Rev. Sci. Inst. **7**, 1079 (1923); **11**, 319 (1925).

L. A. Jones, E. Huse and V. C. Hall, "On the Relation between Time and Intensity in Photographic Exposure," J. Opt. Soc. Am. and Rev. Sci. Inst. **12**, 321 (1926).

L. A. Jones and V. C. Hall, "On the Relation between Time and Intensity in Photographic Exposure," J. Opt. Soc. Am. and Rev. Sci. Inst. **13**, 443 (1926).

L. A. Jones, V. C. Hall, and R. M. Briggs, "On the Relation between Time and Intensity in Photographic Exposure," J. Opt. Soc. Am. and Rev. Sci. Inst. **14**, 223 (1927).

[3] L. A. Jones, "A Special Sensitometer for the Study of the Photographic Reciprocity Law," J. Opt. Soc. Am. and Rev. Sci. Inst. **7**, 305 (1923).

Light Source

The light source used in all of the present measurements was a high intensity, horizontal type, quartz, mercury arc, operated at 48 volts and 2.9 amperes. The arc was placed in a blackened metal lamphouse which could be moved along the lamp track. The lamphouse contained a round window of approximately $1\frac{1}{2}$ inches diameter, directly behind which the arc was mounted. The window was first covered by two (three with the small diaphragm) pieces of ground glass which acted as a diffusing medium. The brass diaphragm limiting the light source was placed in front of the ground glass. In front of the diaphragm was placed the filter for isolating the λ4360 line of the mercury arc. The filter used for this purpose was a Corning glass filter No. 7.1. This filter has a transmission of approximately 10 percent for the λ4360 line, and is opaque to the neighboring lines λ4050 and λ4916, and to all other lines below the red lines of the mercury arc.

In order to test to what extent the red light transmitted by this filter was affecting the exposures made on the high-speed panchromatic emulsion (only one such color sensitized material was used in these tests), a Wratten filter No. 3 was placed in front of the Corning filter, and an exposure of four times that of the normal exposure (i.e., with Corning filter alone) was given to this emulsion. The Wratten filter No. 3, as may be seen from Fig. 2, is opaque to the λ4360 line, but transmits completely the λ4916 line and all lines of longer wave-length of the mercury arc. With this exposure only a slight fogging was obtained on the highest exposure step, and with a normal exposure no fogging was obtained. It was therefore concluded that the only radiation effective in the exposures from the source as used was that from the λ4360 line.

The mercury arc used in this work proved to be a very steady source of radiation when operated in the closed lamphouse as described. The arc was operated from the laboratory storage battery, and to insure against change in voltage during the long exposures an automatic, mechanically operated voltage regulator was employed.

Sector Wheels and Mechanism for Rotating Sector Wheels

The sectors used in these experiments for interrupting the light beam were cut from steel disks 2 mm thick. These sectors were painted a dull black to avoid reflected light. A typical sector is illustrated in Fig. 1. For varying the transmission, $q(=a/b)$, different sectors were available.

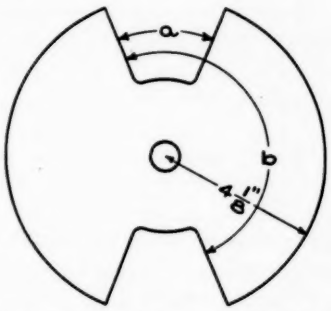

Fig. 1. Sector for interrupting light beam.

In order to vary the frequency of flash, a number of sectors of the same ratio q, having different numbers of blades, were used. Two such sets were available, one for the ratio, $q=1/2$, and the other for the ratio, $q=1/4$. Each set contained sectors having 2, 4, 8, 16 and 32 blades. Hence, with a single speed prime mover, a range of frequencies, one to sixteen, could be covered. To extend this range of frequencies, as was necessary, different speed prime movers were employed. The speeds of the various prime movers used ranged from 1/32 to 150 r.p.s.

Measurement of Lamp Intensity

To measure the absolute intensity of the radiation in the λ4360 line emanating from the light source, a thermopile previously calibrated with a Hefner amyl acetate lamp according to the method given by Coblentz[4] was used. Two Wratten filters, No. 0 and No. 3, were employed in making the intensity measurements of this line. The transmission curves for these filters are shown in Fig. 2. The difference between the galvanometer readings taken with these respec-

[4] W. W. Coblentz, "Measurements on Standards of Radiation in Absolute Value," Bur. Stand. Pap. 11, 87 (1915).

tive filters gave the intensity of the λ4360 line. Care was taken in these measurements to shield the thermopile from the heat radiation of the lamp, a 25 percent solution of $CuSO_4$ in a water cell one centimeter thick being used for absorbing this energy.

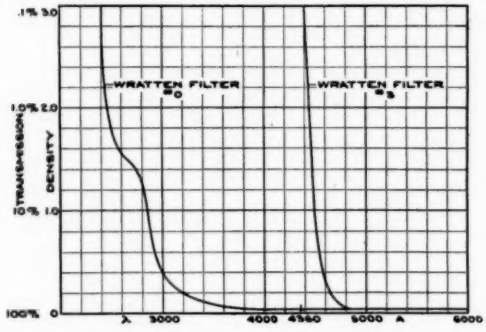

FIG. 2. Transmission curves for Wratten filters No. 0 and No. 3.

The results of these measurements, together with the data from Coblentz's work on the amount of energy per second radiated from the amyl acetate lamp, gave for the value of intensity of the λ4360 line from the mercury arc at a distance of one meter,

$$I = 8.7 \times 10^{10} \text{ quanta/cm}^2\text{/sec.}$$

Throughout this paper values of I are given in terms of quanta per square centimeter per second.

High precision is not claimed for the above intensity measurement. It is believed that the value obtained is within 15 percent of the correct value, and this is sufficiently accurate for the purpose at hand.

EXPOSURES

The exposures were made on film or plates cut to a four by five inch size to fit the reciprocity sensitometer plateholder. Comparison exposures were always made adjacently, or as nearly so as possible, on this single plate or film. This was accomplished by making the exposures through a series of dark slides, which had slots cut at different positions (one slot to a slide), beginning at the edge and extending to the center of the dark slides. This arrangement made possible five exposure curves on one-half of the four by five inch film or plate, and with the dark slides reversed, five additional exposure curves on the remaining half of it. The advantages derived from obtaining comparison data in this work from a single small sample of the emulsion are to be emphasized. This procedure eliminates largely any errors arising from the small irregularities which might occur in emulsion coating, sensitivity, etc., and makes possible the attainment of accurately reproducible results as regards density differences for a series of developments.

DEVELOPMENT

The exposed emulsions were in all cases developed in Pyrogallol developer at 68°F. The composition of this developer is given by the standard Eastman formula D-90, the solutions *A* and *B* being mixed in equal proportions.

A	Sodium bisulfite	17 g
	Pyro	20 g
	Sodium sulfite	70 g
	Water to make	1.0 l
B	Sodium carbonate	75 g
	Potassium bromide	1 g
	Water to make	1 l

The high-speed negative materials used were developed for a period of four minutes, while the slow speed positive material used was developed for only two minutes. Brush development was used throughout the experiments, care being taken to brush the sensitive materials at least once every ten seconds during development. The brush development method[5] is undoubtedly the best means of agitation when uniformity of development over a given area is desired. By this method, differential effects can be accurately reproduced from day to day. This method was therefore adopted as the differential effects were of prime importance in all of these measurements.

DENSITIES

All density measurements were made on a Martens polarization photometer type of visual densitometer.

[5] W. Clark, "Standard Development," Phot. J. 49, 76 (1925).

Results

The first measurements made in these tests were of the magnitude of the reciprocity law failure over a wide range of intensity values. Determinations were subsequently made on the magnitude of the intermittency effect at high

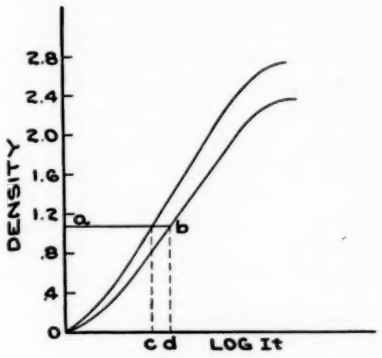

FIG. 3. Characteristic curves.

frequencies of flash, over a similar range of intensities. The method employed for presenting the results of these measurements was suggested largely by the evidence at hand that the intermittency effect is a manifestation of reciprocity law failure. It consists, briefly, of comparing directly the intermittency effect with the reciprocity law failure by plotting the data taken with intermittent exposures, and for average intensity values, along the reciprocity curve. A reciprocity curve exhibits the experimentally determined relationship between the quantities $\log It$ and $\log I$, for a constant density. To obtain such a curve, a number of continuous exposure, time scale, characteristic curves were taken. As an example, two such curves are shown in Fig. 3. A constant density line, as *ab*, was extended horizontally across these curves and the values of $\log It$ (*c* and *d*), required in the two respective cases to produce the stated value of density, were plotted against the corresponding values of $\log I$. The curve thus obtained, when extended over a wide range of values of intensity, exhibits the characteristic appearance illustrated by the curve of Fig. 4.

The four curves labeled I, II, III and IV, shown in Fig. 5, are the experimentally determined reciprocity curves for the four emulsions used in these experiments. The types of photographic materials used are listed below and each has been assigned a number. The numbers of the above curves and the numbers of the emulsions correspond. Hereafter, the emulsions will be referred to by number only.

I. Ordinary blue sensitive, high-speed, negative emulsion.
II. Same type emulsion as I.
III. High-speed, panchromatic, negative emulsion.
IV. Low-speed, blue sensitive, positive emulsion.

Fig. 6 shows the comparison of the intermittency effect with the reciprocity law failure. The solid curves shown in Fig. 6 are the reciprocity curves of Fig. 5 redrawn. The experimental points shown along these solid curves were obtained from *intermittent exposures* and were plotted along the reciprocity curves in order to compare the intermittency effect with the reciprocity law failure over corresponding ranges of intensity. These points were obtained in the following manner. A continuous exposure, time scale, characteristic curve was first taken at the absolute intensity I_1 (see Fig. 4), as in the case of the reciprocity curve. Then a second, intermittent exposure, time scale, characteristic curve was taken at the average intensity value, qI_1, the light beam in this exposure being interrupted f times per second by a sector of transmission q. The values of $\log It$ required to produce a constant density in the two cases were then plotted (by using a common symbol)

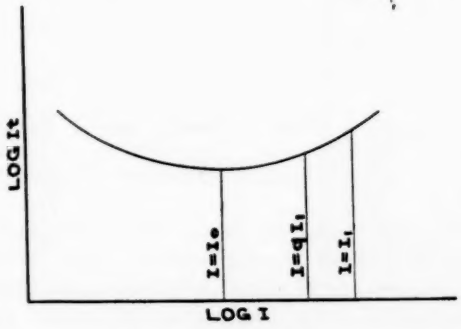

FIG. 4. Reciprocity curve.

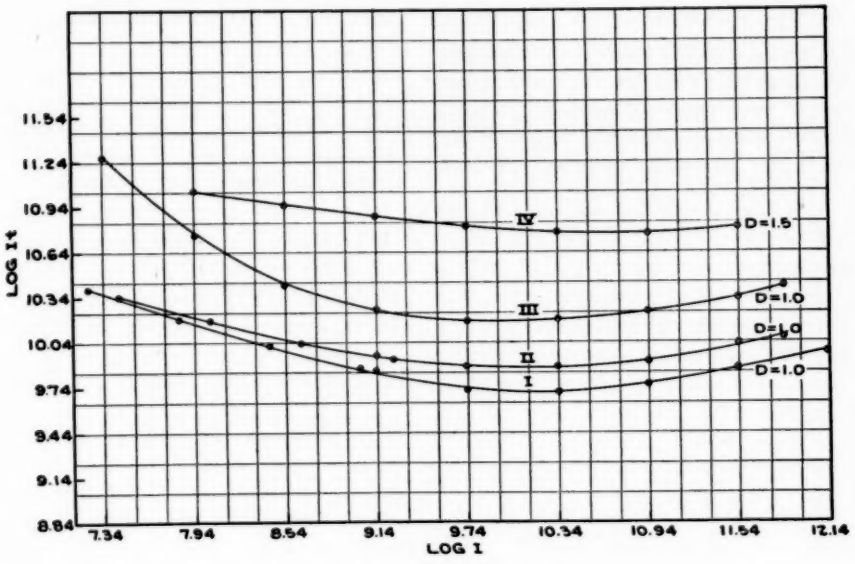

FIG. 5. Reciprocity law failure curves.

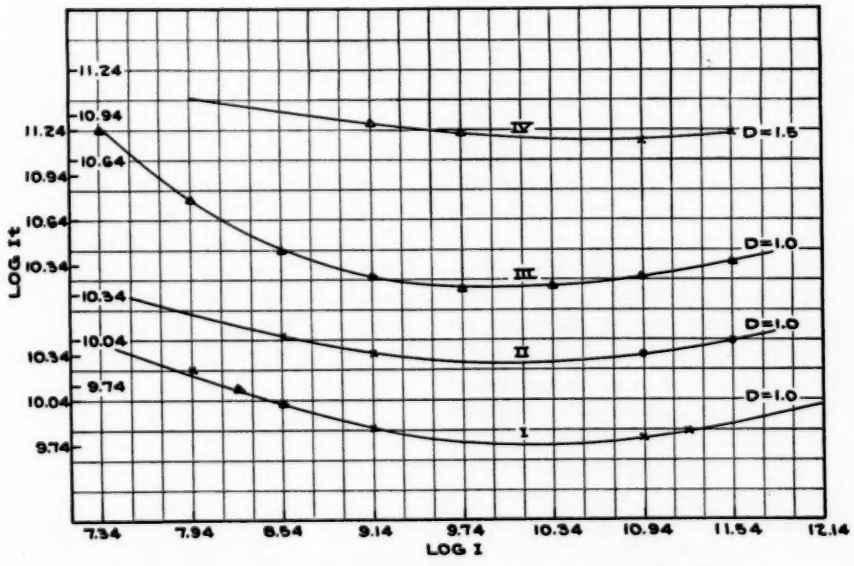

FIG. 6. The intermittency effect compared with the reciprocity law failure.

along the reciprocity curve. The intermittent exposures of all these experiments were made at frequencies of flash well above the predetermined critical values f_c, at which the intermittency effect becomes constant. How this was accomplished will be made clear from the results of the study of the effect of frequency of flash on the intermittency effect, which are to follow.

From the curves of Fig. 6, it may be seen

that intermittent exposures were made for intensity values well above and below optimal intensity for each emulsion tested. For emulsion III, a series of exposures was made extending over the entire range of intensity values of the reciprocity curve. The value of the sector ratio q was maintained at 1/4 throughout the experiments, with the exception of the experiments on emulsion I for which q had the values 1/2 and 1/16. In every case, the experimental points obtained from the intermittent exposures coincided, within the limits of experimental error involved in such measurements, with the reciprocity law failure. The foregoing experimental results may be summarized by the statement that, provided the frequency of flash is above some critical value f_c, an intermittent exposure of average intensity I produces the same photographic density as a continuous exposure of absolute intensity I. Absolute intensity is used here and subsequently to indicate the steady or actual value of intensity in contrast to the average value of intensity.

Variation of Frequency of Flash

The curves of Figs. 7 to 9, inclusive, illustrate the effect of variation of frequency of flash upon the intermittency effect. Curves illustrating the variation of log It with frequency of flash for constant density were drawn for each emulsion. Curves illustrating variation of density with frequency for constant exposure were included for the emulsion IV. This was done because the gamma-value of this emulsion was high, and the variation of density exhibited the larger effect. All of the graphs show that the *main* variation of the intermittency effect took place over a limited range of frequencies, and that the

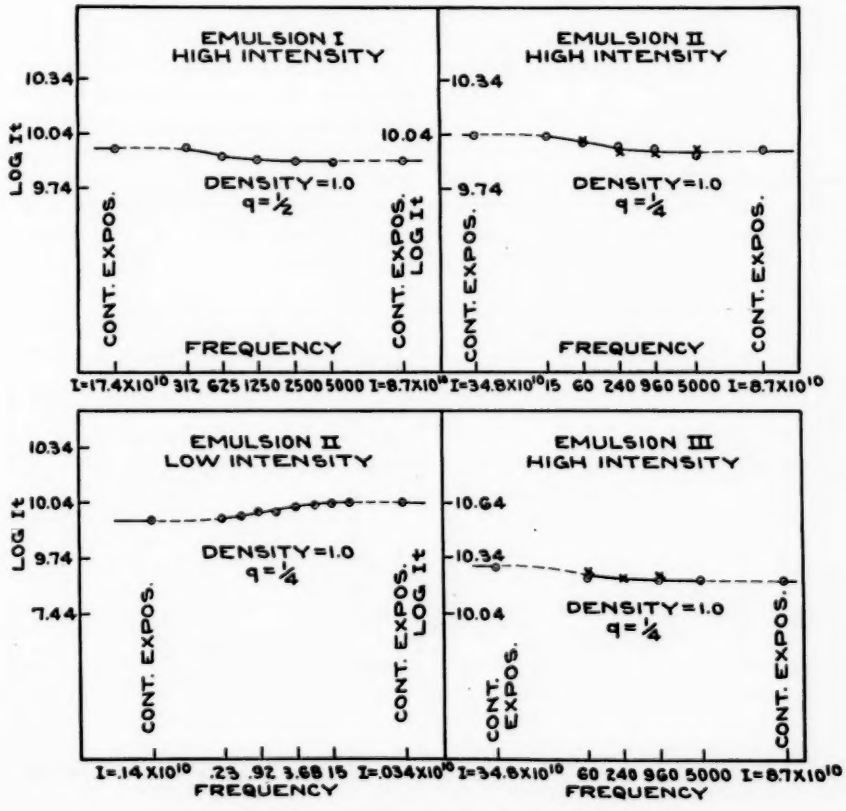

FIG. 7. Effect of frequency of flash on intermittency effect.

variation terminated at a definite critical value f_c. The value of this critical frequency f_c varied with the average value of intensity at which the intermittent exposure was made, as well as with the type of emulsion being tested.

It is to be noted that the vertical shift in the log It values, between zero frequency and the maximum value at which there is no further change, was equal to the magnitude of the reciprocity law failure ($\Delta \log It$) between the two continuous exposures taken at the two levels of intensity, namely, the intensity of the original continuous exposure and the intensity equal to the average intensity of the intermittent exposure.

The curve for the emulsion III, Fig. 8, is to be particularly noted. It shows clearly, because of the large intermittency effect for this emulsion at low intensities, the behavior of the intermittency effect with frequency of flash. The frequency of flash in this experiment was carried well past the critical value f_c in order to show that the intermittency effect remained constant beyond this frequency.

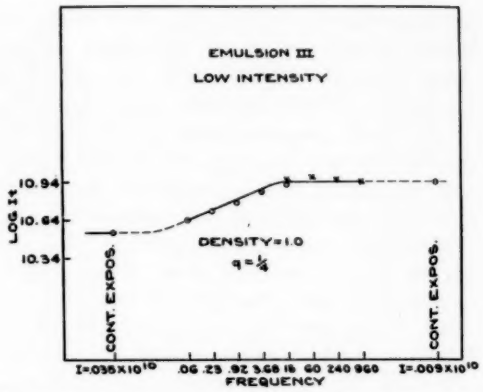

FIG. 8. Effect of frequency of flash on intermittency effect.

DISCUSSION OF RESULTS

The foregoing experimental results show that the intermittency effect becomes, under certain conditions of intensity and frequency, equal in magnitude to the reciprocity law failure. The results afford strong evidence that the intermittency effect, generally, is an aspect of reciprocity law failure, and under special conditions becomes identical with it. From an alternative viewpoint, these results may be interpreted as showing that an intermittent exposure, under particular intensity and frequency conditions, produces the same photographic effect as an equal energy continuous exposure. In the experiments as performed, the reciprocity law failure was invoked as a convenient criterion for determining the conditions under which the photographic effects of two such exposures became equal.

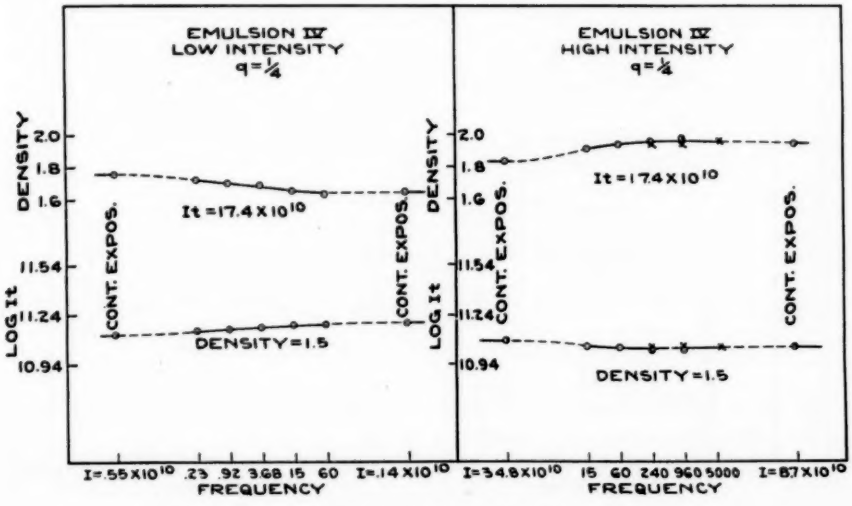

FIG. 9. Effect of frequency of flash on intermittency effect

Considered from the standpoint of discrete light quanta incident upon a photographic grain, all exposures may be viewed as intermittent exposures. Further it is known that reciprocity law failure is governed in some manner, yet undetermined, by the time distribution of reception of quanta by some portion or all of a photographic grain. Indeed, such a statement practically constitutes the definition of reciprocity law failure.

In light of these facts, the simplest and most direct explanation of the above experimental results seems to consist in the fact, subsequently to be demonstrated, that the time distribution of the reception of quanta in an intermittent exposure becomes, under certain intensity and frequency conditions, substantially identical with that in an equal energy continuous exposure. The obvious reasoning here involved is that the two types of exposure, under the special conditions outlined, would become indistinguishable to the photographic grain. Such an explanation is seen to be concerned entirely with the comparison of the time distributions of quantum incidences on the photographic grain in the two types of exposure under consideration, and is not concerned in any manner with the mechanism of reciprocity law failure. The conditions of intensity and frequency for which the time distribution of quanta in the two types of exposure become equal may be seen from the following qualitative argument.

In a continuous exposure of absolute intensity I, the time separation of successive hits by quanta on the effective receptive area ρ of a photographic grain is a random phenomenon. From a statistical consideration it can be shown that the times of separation will be distributed about the mean time $\bar{t}$ in the same manner as, in the mathematically analogous problem of molecular motion, the molecular paths are distributed about the mean path. By analogy, if there is a total number of quanta N incident upon an area ρ, then the number of successive incidences separated by an interval between t and $t+dt$ is given by the expression,[6]

$$dn = Ne^{-t/\bar{t}}dt/t. \tag{1}$$

The quantity dn can be expressed as an explicit function of $\log t/\bar{t}$ and in this form is better suited for the present study. The quantity dn, viewed as a function of $\log t/\bar{t}$, takes on the general Gaussian shape, and exhibits a maximum at the point for which $t=\bar{t}$. With this change in form, (1) becomes,

$$dn = Ne^{-e^{\log t/\bar{t}}e^{\log t/\bar{t}}}d(\log t/\bar{t}). \tag{2}$$

A graph of the quantity $dn/d(\log t/\bar{t})$, for $\bar{t}=1$, Fig. 10, shows the time distribution about the mean time $\bar{t}$ of the reception of quanta in the continuous exposure under consideration.

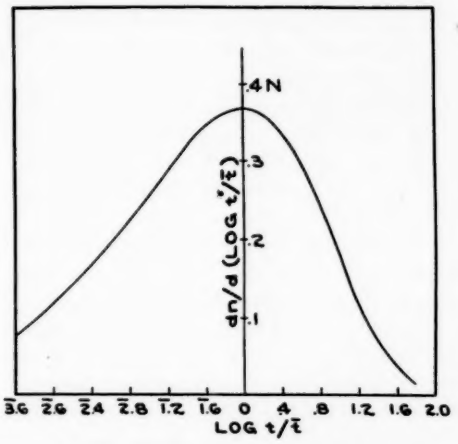

FIG. 10. Distribution of quanta in time.

Consider an intermittent exposure of average intensity equal to that of the above continuous exposure and assume the frequency of flash of this intermittent exposure raised to a value such that there is, on the average, only one quantum per flash incident on the area ρ. The time distribution of reception of quanta in such an exposure will approach the same shape as that of the continuous exposure illustrated by the curve of Fig. 10. The distribution curves will differ somewhat in form[7] because of the ratio q

[6] Leonard B. Loeb, *Nature of a Gas*, J. Wiley & Son, New York, 1931, p. 68.
Leigh Page, *Introduction to Theoretical Physics*, D. Van Nostrand, New York, 1st edition, 1928, p. 306.

[7] Consistent small experimental discrepancies between the effects of intermittent and continuous exposures have been reported by E. A. Baker (reference 1) and B. O'Brien (reference 1) and it is thought that these discrepancies are attributable to this cause.

of the sector used and the consequent selective effect introduced. However, as the time between flashes in such an intermittent exposure equals the average time between quantum incidences, in the continuous exposure the main peaks of the two distribution curves will coincide and the general shapes of the curves will substantially agree.

If the reciprocity law failure and the intermittency effect alike are caused purely by the variation in the time distribution of the reception of quanta, then it is seen that the above continuous and intermittent exposures should produce the same photographic effects. This explanation seems to be consistent in every way with the experimental result found, that above a certain critical frequency f_c, the intermittent and continuous exposures produce the same photographic effect.

It can be shown that this explanation is in accord with the other experimental facts observed that the value of the frequency f_c, at which the intermittency effect becomes constant and equal to the reciprocity law failure, varies with the average intensity of the intermittent exposure and with the type of emulsion. At constant absolute intensity I_{ab}, the frequency of flash required to allow but one quantum per flash on unit area is proportional to the sector ratio q, while at constant sector ratio q, the frequency required to allow but one quantum per flash on unit area is proportional to the absolute intensity I_{ab}. Therefore, for both q and I_{ab} variable, the frequency necessary to allow but one quantum per flash on unit area is proportional to the product of I_{ab} and q. As this product gives the number of quanta incident per square centimeter per second, it is seen that the frequency required to allow but one quantum per flash on an area ρ square centimeters is given by the equation

$$f = \rho(I_{ab} x q) \equiv \rho I_{av}. \qquad (3)$$

This equation expresses the fact that f is directly proportional to the size of the projective area ρ and to the average intensity I_{av}. If in Eq. (3) the area ρ be identified with the area on a photographic grain effective in receiving quanta in exposure, and the frequency f be identified with the critical frequency f_c, found experimentally, at which the intermittency effect becomes constant and equal in magnitude to the reciprocity law failure, then Eq. (3) predicts the direct dependence of f_c upon ρ and I_{av} in accordance with experimental fact.

Viewed conversely, Eq. (3) affords a means for determining the size of the effective receptive area ρ of the photographic grain when values of f_c and I_{av} are known. Values of ρ thus calculated, by using the experimentally determined values of f_c and I_{av}, are given in Table I. The significant

TABLE I. *Size of area on photographic grain effective in receiving quanta in exposure.*

Fig.	Emulsion	f_c	I_{av}	Effective receptive area ρ	Average grain size observed
7	I	1250	8.7×10^{10}	1.4 μ^2	$0.7\mu^2$
7	II	7.5	0.034	2.2 μ^2	
7	II	960	8.7	1.1 μ^2	
7	III	960	8.7×10^{10}	1.1 μ^2	$0.5\mu^2$
8	III	5	0.0085	6 μ^2	
9	IV	15	0.14×10^{10}	1.1 μ^2	$0.1\mu^2$
9	IV	240	8.7	$0.27\mu^2$	

fact to be noted from Table I is that ρ, in size, is found to be of the order of magnitude of that of the entire photographic grain (or perhaps small clumps of grains), rather than of a fractional area of the grain. This indicates strongly that the entire photographic grain is the effective receptor of quanta in photographic exposure. That the calculated values of ρ are larger than the average grain sizes may be understood by noting that if the entire projective area of the grain receives the quanta, then f_c and hence the calculated values of ρ must be large enough to conform to the largest grains occurring in any appreciable numbers in the emulsion. An inspection of the grain size frequency charts of Figs. 11 (Emulsion I and II), 12 (Emulsion III), and 13 (Emulsion IV) shows that considerable area is covered by grains of sizes corresponding to the calculated values of ρ. In the case of each emulsion, the values of ρ calculated for the low intensities were larger than those calculated for high intensities. This is undoubtedly significant of the fact, determined by previous experiments, that there is a preponderance of large

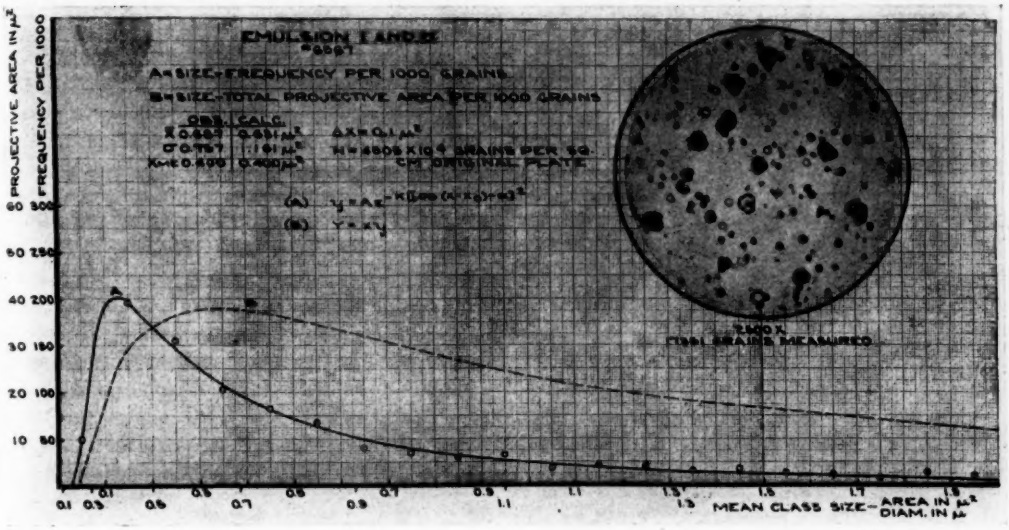

FIG. 11. Grain size frequency distribution for Emulsions I and II.

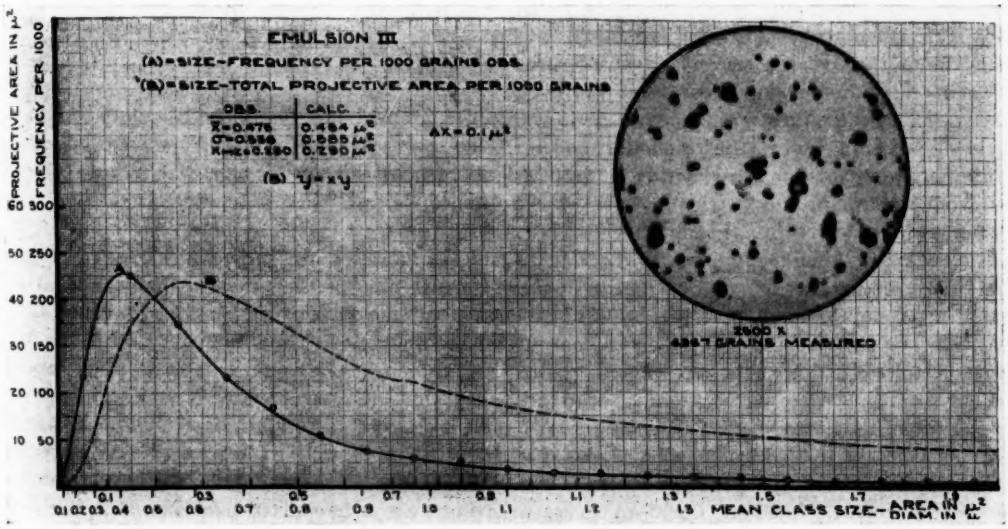

FIG. 12. Grain size frequency distribution for Emulsion III.

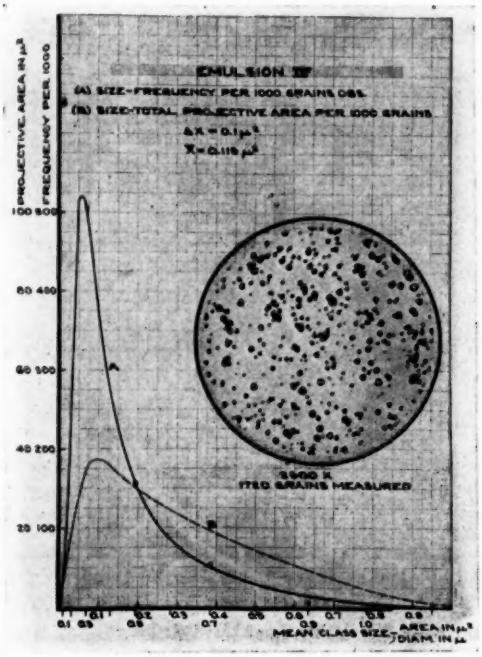

FIG. 13. Grain size frequency distribution for Emulsion IV.

grains made developable by exposures at low intensities. In this connection it should be noted that the calculated values of ρ should not conform to the average size of projective area of grains in the undeveloped emulsion, nor even necessarily to the largest grains occurring in the undeveloped emulsion. Rather, ρ should conform to the largest grains, or areas, acting as units in receiving quanta in the particular type of exposure under consideration. From this viewpoint, it is clear that ρ should more nearly conform to the largest size grains or units occurring in the image after development. On the basis of the sensitivity speck theory, ρ would conform to the largest areas acting as the receivers of quanta per sensitivity center active alone in the development of that area.

An interesting further calculation shows, for some of the continuous exposures made, the number of quanta incident upon an average sized photographic grain, and hence the order of magnitude of the number of quanta effective in producing developability of an average sized grain. The results of this calculation are given in Table II.

TABLE II. *Number of quanta required to produce developability of photographic grain.* (Density = 1.0.)

Emulsion	Exposure time t	Average grain size σ	Intensity I	$N=\sigma It$
I	1.0 sec.	$0.7\mu^2$	0.55×10^{10}	38
II	1.5	$0.7\mu^2$	0.55	58
III	2.5	$0.5\mu^2$	0.55	69
IV	12	$0.1\mu^2$	0.56	66

Table III contains the results of another calculation which are thought to be of interest. The rates of reception of quanta by an average sized grain for some of the low intensity exposures are given. The values of P, in column five, give the average number of quantum incidences per second per average sized grain.

TABLE III. *Rate of reception of quanta by photographic grains in low intensity exposures.* (Density = 1.0.)

Emulsion	Exposure time t	Average grain size σ	Intensity I	Quanta / Grain / sec. $P=\sigma I$	Average time between quantum incidences $1/P$
I	22 min.	$0.7\mu^2$	0.0022×10^{10}	0.154	6.5 sec.
II	17	$0.7\mu^2$	0.0022	0.154	6.5
III	68	$0.5\mu^2$	0.0022	0.11	9
IV	45	$0.1\mu^2$	0.009	0.09	11

In column six are given the calculated values of the average time elapsing between quantum incidences for an average size grain. The remarkable fact to be noted here is the extremely low rate of reception of energy by a single grain of the emulsion. In these exposures, there is an elapsed time of the order of magnitude of five to ten seconds between quantum incidences, an exceedingly long time compared with the time of any single electronic or atomic phenomenon. Yet this low rate of energy reception is sufficiently great to produce a strong photographic density if allowed to act for the time shown. Other exposures have been made, though the results are not reported here, in which the average time between quantum incidences was greater than in the present exposures, in some cases amounting to eighty seconds. In these cases, it was still possible to obtain strong images.

Summary

(1) A comparison of the magnitudes of reciprocity law failure and the intermittency effect, by using monochromatic radiation of wave-length 4360A, was made for four different emulsions.

(2) It was found that, providing the frequency of flash was above a critical value f_c, an intermittent exposure of average intensity I produced the same photographic effect as a continuous exposure of absolute intensity I.

(3) The critical value of frequency f_c was found to be different for different emulsions and to vary directly with the average intensity of the intermittent exposure.

(4) A theoretical explanation of the above experimental results, based on a quantum theory of exposure, yielded the formula, $f_c = \rho I_{av}$, in which ρ is the area on a photographic grain effective in receiving quanta, and I_{av} is the average intensity of the intermittent exposure. By means of this equation, the values of ρ for the different emulsions were calculated. The values found showed ρ to be of the order of magnitude of the size of the photographic grain.

Conclusions

(1) From the experimental results of this work, it appears that the intermittency effect is a manifestation of reciprocity law failure.

(2) The same photographic effect is obtained with an intermittent exposure of average intensity I as with a continuous exposure of absolute intensity I, providing the frequency of flash of the intermittent exposure is above a minimum critical value f_c.

(3) The critical frequency f_c of an intermittent exposure is that frequency at which, on the average, there is only one quantum per flash incident on the effective receptive area ρ of the photographic grain.

(4) The area ρ of a photographic grain, effective in receiving quanta during photographic exposure, is of the order of magnitude of the size of the photographic grain and may be calculated from the equation $f_c = \rho I_{av}$, in which I_{av} is the average intensity of the intermittent exposure.

Light Intensity at Different Depths in Lake Water

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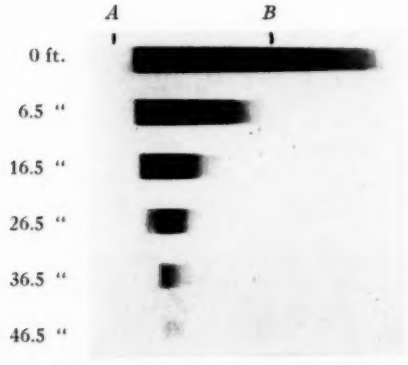
(Received December 19, 1932)

A quartz prism dispersing system in a specially designed housing was used for evaluating the intensities of different wave-lengths in the visible spectrum at different depths in water on Gunflint Lake on the northern border of Minnesota. The spectrum was photographed at different points down to 70 ft. and from these the intensities of the different wave-lengths were evaluated from characteristic curves obtained from standard exposures by using wire screens. It was found that the variation of the intensity with depth was exponential in the case of some wave-lengths but not in the case of all. From the results obtained, absorption constants were obtained for the different wave-lengths.

THE object in this investigation was to determine the feasibility of using a quartz spectrograph combined with the photographic method for determining the variation with depth in water, of the intensities of the different wave lengths of light, under the natural conditions such as prevail in our fresh water lakes and oceans.

By using a dispersing system, a spectrum of the light present at any point in the water is obtained, thus rendering apparent the transmitting power of water for the different wave-lengths.

In Fig. 1 are shown a number of spectra obtained by means of a quartz spectrograph at the different depths there indicated, the times of exposure being 30 seconds in the case of each.



AB=Visible Spectrum.
FIG. 1. Showing the shortening of the light spectrum at different depths, using 30 second exposure.

It is there seen how the spectrum shortens at both ends, showing a marked absorption by the water of the longer and shorter wave-lengths. It is also seen that at a depth of 46.5 ft. only the portion in the yellow region of the visible part of the spectrum is present with sufficient intensity to perceptibly affect the photographic plate in the 30 sec. of exposure.

The marked absorption of the lake water for the longer and shorter wave-lengths with a region of greater transmission for the intermediate wave-lengths is in accord with Beebe's[3, 17] observations with a Zeiss visual spectroscope made during his remarkable descent into the ocean. There is a difference, however, in that in the case of the ocean water Beebe found that at a depth of 800 feet the spectral region of greatest transparency was for blue light whereas in the case of the water of Gunflint Lake at a depth of 46.5 feet the region of the greatest transparency was for yellow light. The difference may probably be attributed to differences of the impurities, animal, vegetable, or mineral, in the ocean water and the lake water.

In this work an attempt has been made to obtain a quantitative comparison of the intensities of the different parts of the spectrum as the depth is increased.

The ensemble intensity of the light present at different depths has been determined with considerable care, by using photoelectric cells[35, 38] and thermocouples.[4–11]

Apparatus

The quartz spectrograph used included the usual optical system consisting of a slit, collimator lens, cornu 60° prism, telescope lens and photographic plateholder for a 4 3/16″×7″ plate. This optical system was enclosed by a specially designed aluminum housing as shown in Fig. 2.

A quartz plano-convex lens 4 cm in diameter was placed at *A*, Fig. 2, 20 cm from the slit *C*. The inner plane surface of the lens at *A* was ground for diffusing the light. This lens was sealed into the tube *B* by means of marine glue. The tube *B* served also as a sealing cap for the end of the collimator tube *D*.

An Eastman shutter *E* was placed immediately in front of the slit *C*. The shutter *E* was opened and closed by means of a fine optician's chain passing through a tube to an electromagnet *F* placed in the housing *G*. The open and closed positions of the shutter were indicated on a galvanometer, *H*, through a circuit in which advantage was taken of the partial return of the shutter lever when in the open position. The galvanometer circuit contained three flash light cells, *I*, which discharged through two megohms,

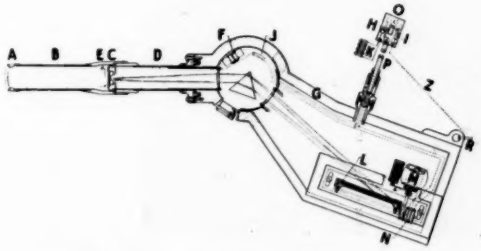


Fig. 2. Showing diagram of quartz spectrograph.

J, on open shutter and one megohm on closed shutter, contact to ground on closed shutter being made through the shutter lever. A diagram of shutter and indicator circuits is shown in Fig. 2. The shutter was operated by means of a 40 volt dry cell battery *K* and required 3.5 amperes during the short interval required for opening and closing.

Between exposures, the plateholder, *L*, was shifted by means of the electric motor, *M*. The length of the plate used permitted 21 exposures each 4 mm wide, separated by spaces 4.5 mm wide.

The motor, *M*, was operated by a double throw switch and the same battery, *K*, as for the shutter, and required 3.5 amperes during the short interval required for shifting the plate. A diagram of the plate circuit is given in Fig. 2.

The plate circuit was closed through a commutator, *N*, which opened the circuit at the end of the desired travel of the plateholder and also at that instant closed a parallel circuit in preparation for the next shift. The motor, *M*, moved the plateholder through a train of gears and worm gear as shown in Fig. 2.

The electric meters and keys required for operating the plateholder, shutter and shutter indicator, were contained in a case, *O*, specially designed for convenience in operation. The three conductors required for the above operation composed a triplex rubber covered cable, *P*. The steel cable, *R*, required for holding the spectrograph in position in the water served also as a common return as ground circuit, see Fig. 2.

The operation position is shown in Fig. 3. In order to hold the boat in position, two buoys were placed about 100 ft. apart, each buoy being held in position by three 3/16 inch steel cables secured to rock anchors placed in a triangle as shown in Fig. 3. The boat was held in position by a steel cable about 100 ft. long, connecting the buoys. The direction of this cable was N.W. by S.E.

In order to hold the spectrograph in the desired position a specially designed pulley anchor, was placed on the bottom on the south side of the cross cable. The anchor contained a pulley 5 inches in diameter, supported on a metal base and was provided with a trip lever which could be operated by the trip cable, Fig. 3. The metal anchor was placed on and attached to a wooden platform four feet square in order to prevent settling. A heavy flat rock was secured to the bottom of this platform in order to counteract with a sufficient margin the buoyancy of the spectrograph and the pull in the cables.

A 3/16 inch steel cable, having 1/2 inch brass nubbins 10 feet apart, was attached to the spectrograph and passed under the pulley of the anchor. This cable was continued to the boat containing the observers, where it passed through

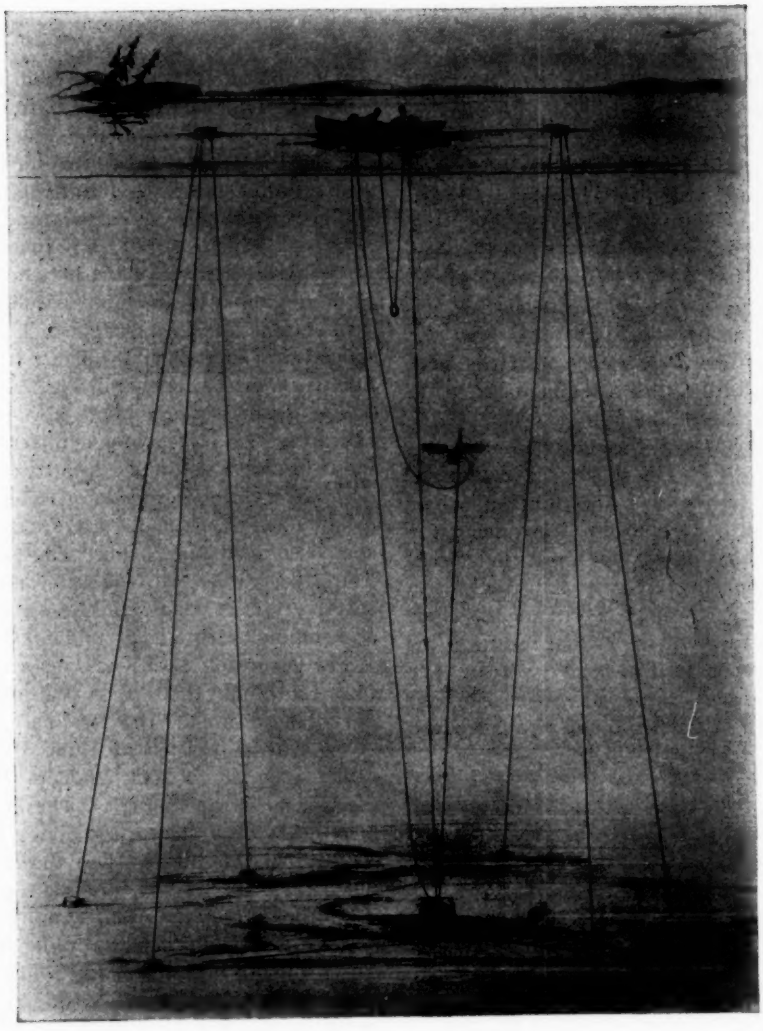


FIG. 3. Showing arrangement for holding spectrograph and observers in position.

a ring and down through a heary iron torus and back to the boat where it was connected electrically so as to also serve as the return circuit.

By means of the nubbins cable, the spectrograph was pulled down and held in the desired position beneath the water surface, the brass nubbin engaging the pawl in the anchor at the bottom. This placement gave the spectrograph the steadiness essential for proper exposure.

As the weight of the spectrograph was 95 lbs. and being heavier than the water displaced, two floats were attached to it as shown in Fig. 3. Each of these had a volume of 7/8 cu./ft. and were constructed of sugar pine covered in a water tight manner by soldered 6 oz. sheet copper. As sugar pine has a density of 21 lbs./cu. ft. the two floats furnished a buoyant force of about 70 lbs. which was sufficient to float the

spectrograph with a residual buoyancy of about 12 lbs. when completely immersed.

It was thus possible to place the spectrograph with the collimator tube vertical and at the various depths determined by the brass nubbins on the nubbins cable.

The procedure was to place the spectrograph just beneath the water surface, open the shutter by pressing the shutter key in the instrument box *O*, waiting the predetermined time, and again pressing the shutter key.

By pressing the galvanometer key in the instrument case, the proper functioning of the shutter could be determined.

Having obtained the exposure at the surface the spectrograph was pulled down to the position determined by the next nubbin on the cable, the plate shifted and the exposure repeated, during the desired time.

In the case of this work the depth of the water was 150 ft. On account of the calibration limitations, the 150 ft. range has to be covered in three sections, namely 0–50 ft., 50–100 ft., and 100–150 ft. This permits on the 7 inch plate an ample series of exposures not only on the down journey but also on the up journey, thus giving two series of exposures on the same plate.

The approximate length of exposure required at the different depths was determined by obtaining test exposures before the final exposures were obtained. The times selected were those which gave a blackening on the plate which was well within the range of the blackening *vs.* log-intensity curve, i.e., the so-called characteristic curve, as determined by the calibration with the direct sun light and wire darkening screens of known factors.

Photometric Procedure[15]

Having obtained the exposures at the different depths the next step was to obtain on the same plate a series of exposures from which the characteristic curve for the times involved, could be constructed.

The plate having the exposures to be evaluated on one-half was placed in the holder so that the other half could be exposed. A series of six or seven exposures were then obtained, by using direct sunlight and wire screens of known reduction factors. One series was obtained for each of the times used in the field exposures.

In a range of 50 ft. it was found that three different times were ample thus requiring three different characteristic curves. The twenty-one exposures possible on one-half of the 7 inch plate permitted the three series to be obtained. The times required for the range 0 to 50 ft. were 2 sec., 30 sec. and 60 sec.

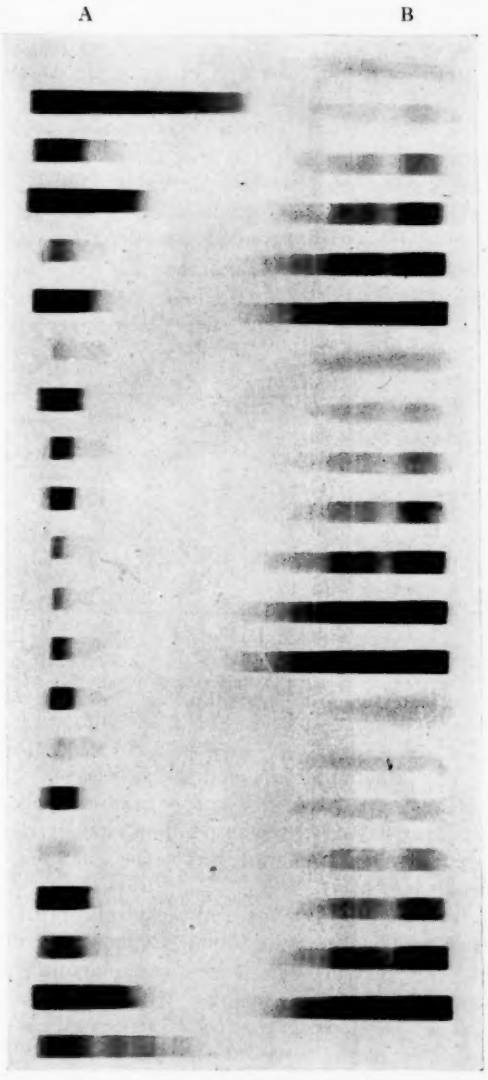


Fig. 4. Showing a sample plate giving field exposures and calibration exposures.

Obtaining calibration exposures on the same plate as the field exposures insures the same conditions as regards emulsion, development, etc. A series of exposures including the calibration exposures are shown in Fig. 4. The series *A* are the field exposures and *B* the calibration exposures.

The plate thus obtained was then developed and the intensity of the blackening for a given wave-length measured on the microphotometer.

In the case of each calibration series, the blackening was plotted as ordinates against the logarithm of the intensity, by using the surface intensity as unity and the proper screen factor. The characteristic curves corresponding to the wave-length 5450A, for the three calibration series given on the plate shown in Fig. 4, are given in Fig. 5. The total sunlight intensity at the time of each exposure was determined by means of a photronic illuminator.

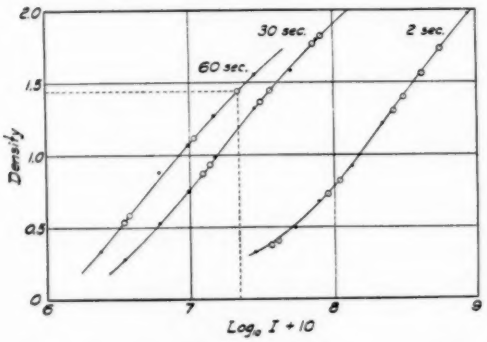


FIG. 5. Showing the three characteristic curves obtained from plate shown in Fig. 4, and location of the field exposures.

The microphotometer reading giving the blackening on a field exposure was reduced to the intensity of the corresponding characteristic curve by extrapolation from the illuminator readings.

The log intensity for each field exposure was then obtained from the points of equal blackening on the characteristic curve corresponding to same time and wave-length.

The location of the field intensities corresponding to the wave-length 5450A on the field exposures shown in Fig. 4 are shown by the circles in Fig. 5.

In Table I are given the intensities obtained at the different depths in the case of the wave-lengths selected for evaluation. The wave-length

TABLE I.

Depth	Orange 6230A Intensity	Av. No.	Orange & Yellow 5810A Intensity	Av. No.	Yellow 5630A Intensity	Av. No.	Yellow & Green 5450A Intensity	Av. No.
0 Ft.	1.000	7	1.0000	7	1.0000	7	1.0000	7
6.5 Ft.	0.506	7	0.4920	6	0.5550	6	0.489	6
16.5 Ft.	0.140	12	0.2030	14	0.2330	14	0.1553	13
26.5 Ft.	0.0408	8	0.0913	14	0.0860	13	0.0584	12
36.5 Ft.	0.0104	16	0.0350	16	0.0323	16	0.0187	16
46.5 Ft.			0.01272	10	0.0101	10	0.0059	10

Depth	Green 5290A Intensity	Av. No.	Green & Blue 5000A Intensity	Av. No.	Blue 4660A Intensity	Av. No.	Violet 4470A Intensity	Av. No.
0 Ft.	1.0000	7	1.0000	7	1.0000	7	1.0000	7
6.5 Ft.	0.4600	6	0.3351	7	0.245	7	0.1330	7
16.5 Ft.	0.1448	14	0.0805	11	0.0221	7	0.00497	6
26.5 Ft.	0.0462	11	0.0156	7	0.0050	1		
36.5 Ft.	0.0124	16	0.0039	3				
46.5 Ft.	0.0074	6						

and corresponding color region are given at the top. The intensities evaluated are, as is seen, on the basis of unit intensity at the surface and represent the average of the number of observations given at the right of each value. In Fig. 6 is given the graphs of the log of these intensities against the depth. It is seen that the decrease in intensity is quite exponential. The greatest deviation from exponentiality seems to be in the case of wave-lengths 5630A and 5290A. The indications are that certain wave-length regions deviate from the exponential relationship.

In Table II are given the absorption constants for the different wave-lengths as determined

TABLE II.

Wave-length	6230A	5810A	5630A	5450A	5290A	5000A	4660A	4470A
y	0.4070	0.3087	0.322	0.357	0.394	0.501	0.740	1.046

from the graphs in Fig. 6. These constants represent the constant y in the equation:

$$I = I_0 e^{-\gamma D},$$

where D is the depth in meters, I_0 the intensity

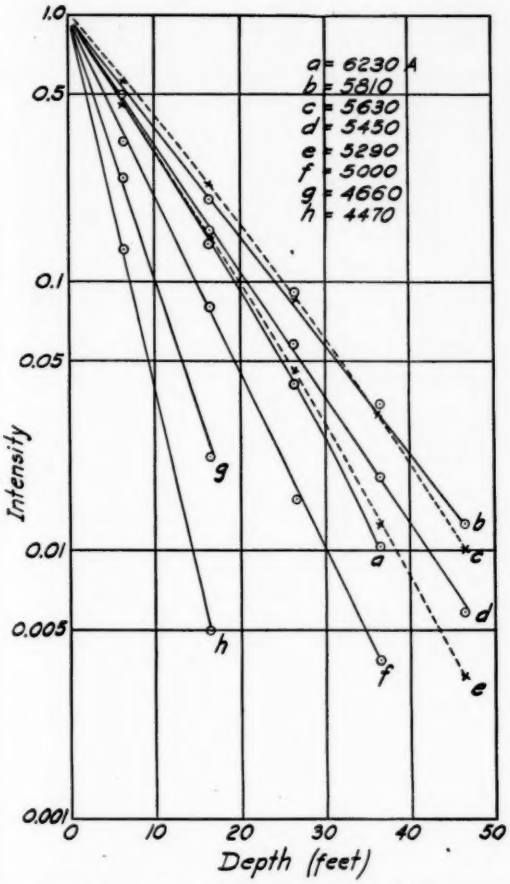


FIG. 6. Showing relation of light intensity to depth in water.

at the surface and I the intensity at the depth D. As the dimension of the constant y is the reciprocal of length only, the values given in Table II may be used for calculating the intensity at any depth if the intensity I_0 at the surface is known.

In Fig. 7 is shown the curves obtained by plotting intensity at the different depths against the wave-length. It is seen that the maximum intensity at the different depths corresponds to a wave-length of the order of 5700A, i.e., the yellow, there being a gradual shift towards the longer wave-length as the depth increases. Results obtained, however, in the Pacific waters at Hawaii seem to indicate that the maximum is in the blue region which is quite in accord with Beebe's observations.

TEST MEASUREMENTS

In order to test the degree of accuracy of the above procedure, measurements were carried out in the laboratory with a 500 watt Mazda lamp and making use of the calculated inverse square intensities as checks.

Exposures were made without screens when the lamp was placed at different distances and using times of exposure which gave blackenings which were suitable for intensity evaluation from the characteristic curves.

Exposures were then made on the same plate with sunlight and suitable wire screens of known factors. As seven exposures are required for each characteristic curve, two of these were obtained with times corresponding to the 10 sec. and 5 minutes used in the field exposures. The results obtained are given in Table III. It is seen that beyond two and one-half meters the check is of a very satisfactory order.

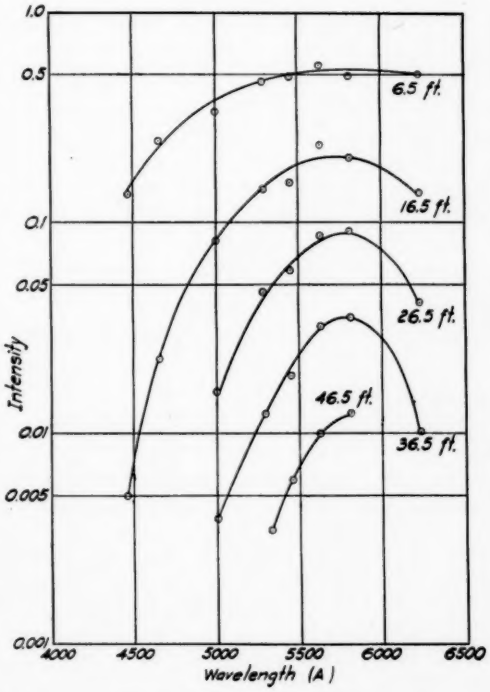


FIG. 7. Showing relation of intensity and wave-length at different depths.

TABLE III.

Distance in meters	$1/D2$	Intensity observed
1.61	0.3870	0.3000
2.02	0.2450	0.2075
2.41	0.1720	0.1563
3.08	0.1050	0.1042
3.92	0.0650	0.0641
5.22	0.0368	0.0364
6.41	0.0243	0.0246
8.65	0.0134	0.0133
10.58	0.00895	0.00921
11.87	0.0071	0.00693

The above observations were obtained during the last two weeks of August, 1932. During this time every suitable day for observations was utilized and the first 50 feet consumed the entire time. Because of limitations in time and bad weather conditions, intensities below 70 feet were not obtained although everything was set for the full range of 150 feet. It was estimated that at 100 feet a length of exposure of the order of two hours would be necessary.

A record of the temperature at the different depths is given in Fig. 8. The temperature relations there given remained virtually unchanged during the time of the above work.

At a later time an attempt will be made to determine what effect temperature has upon the photographic exposures in order to determine if a correction should be applied to the coefficients given above. This test may be conveniently carried out in the laboratory.

CONCLUSION

The experience gained in the work thus far and the results obtained indicate that the method employed is feasible.

It seems clear, however, that the problem should be attacked under more favorable conditions with reference to facilities which will permit independence of unfavorable wind conditions and which will render the attainment of results more expeditious. In addition, a correlation should be sought between the light intensities of the different portions of the spectrum and the suspensions in the way of plankton and organic content. With this in view the work should be carried on cooperatively with those having an interest in the biological aspect. Several summer seasons might well be devoted to this work. Waters of the highest clarity as well as waters having a high plankton content should be studied in detail.

The writer wishes to express his appreciation of the efficient help rendered in this work by his assistant, Mr. Alfred Nier.

The writer also wishes to express his gratitude to Mr. Charles Pillsbury, Mr. Alfred Pillsbury, Mr. A. P. Anderson and Mr. James Ford Bell for financial assistance which enabled this work to get underway during an expedition to Hawaii.

An expression of appreciation is also due the Executive Committee of the Graduate School at the University of Minnesota for the financial aid which made the later work possible.

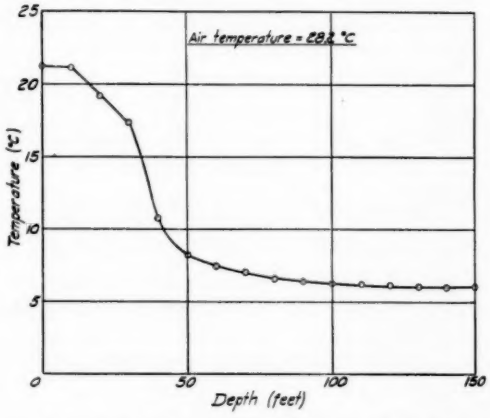


FIG. 8. Showing temperatures of water at different depths in Gun Flint Lake in Minnesota on August 16, 1932 at 2 P.M.

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Focus of a Concave Grating Spectrograph

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The problem of mounting a twenty-one foot concave grating and slit at oblique incidence, so that the focus will lie on a previously established track having less than one inch adjustment range, is discussed. A simple technique, employing only a good transit, is described. The development of this method depends on a precise knowledge of the nature of the general focal curve. This information is found by a brief examination of grating theory. A table of permissible departures of the slit from the Rowland circle is given, such that the error in path length to the sharpest image is less than $\lambda/4$. Application of classical grating theory to the case of oblique incidence and diffraction results in an expression for maximum useful grating width. This expression is shown to differ only by a factor of 1.06 from the expression derived in a recent article by Mack, Stehn and Edlén.

THE problem of mounting a twenty-one foot concave grating with the slit at 45° from the normal, to be used on the same circle with another grating previously focussed at normal incidence, was recently encountered in the spectroscopy laboratory of the Massachusetts Institute of Technology. The track adjustments could not be disturbed because of its use with the original grating. Plateholders for use with the new grating were built with 3/4 inch adjustment ranges. These were clamped to the fixed track.

With the circle of focus established within this region of less than an inch in twenty-one feet, the problem of focussing was an unusual one. It became especially troublesome when the focal curve failed to behave in any anticipated manner when the slit or grating was moved. It was thought that the general focal curve was an ellipse tangent to the grating center, passing through the slit, and symmetrical about the normal to the center of the grating. The following statement given by Mack, Stehn and Edlén[1] appeared to be another simple and reasonable description of the focal curve for the case of the slit not on the Rowland circle:

"By the usual methods of adjustment, the photographic plate is brought quite accurately into focus with the slit; i.e., *both are on a circle of radius* $\rho/2$ (ρ = radius of curvature of the grating) *tangent to the grating circle. The common tangent, however, is not necessarily the one parallel to the axis along which the constant grating interval* σ *is measured.* Let us consider the effect of a small displacement of the point of tangency, *as though the Rowland circle were rolled along the grating circle.* Calculating the path in terms of the ordinate measured from the center of the grating *we find terms of every power in* y."[2]

When neither of these concepts led to progress the author reviewed the concise, but quite complete treatment of grating theory given by Schuster in his treatise, *The Theory of Optics.* The easy extension of this treatment which follows leads to the desired information as to the behavior of the general focal curve.

We use the same notation (Fig. 1) as Mack, Stehn and Edlén, with the addition of the Cartesian coordinates (a, b) of the slit, (a', b') of the point A' to which the path length is being calculated, $u = PA$, and $v = PA'$. Then using:

$$a^2+b^2=r^2, \qquad 2\rho x=x^2+y^2,$$

[1] Mack, Stehn and Edlén, "On the Concave Grating Spectrograph," J. Opt. Soc. Am. **22**, 260 (1932).

[2] A communication from Dr. Mack advises that the paragraph quoted was not intended to be a discussion of the general focal curve. According to that author, the quoted paragraph should begin somewhat as follows—"By the Upsala method of adjustment, the photographic plate and the slit are placed quite accurately on a circle of radius $\rho/2$, tangent to the grating circle." The remainder of the paragraph then discusses correctly the errors in phase introduced by the special misadjustment described there.

we find

$$u^2=(x-a)^2+(y-b)^2=r^2-2by+y^2+x^2-2ax=(r-by/r)^2+(a/r^2-1/\rho)ay^2+(1-a/\rho)x^2.$$

The second term is of the second order of magnitude in y, and the third term is of fourth order in y. Neglecting fourth order terms temporarily, we find:

$$u=(r-by/r)+\tfrac{1}{2}\frac{(a/r^2-1/\rho)ay^2}{(r-by/r)}+\cdots=(r-by/r)+\frac{1}{2r}(a/r^2-1/\rho)ay^2(1+by/r^2+\cdots)+\cdots.$$

Similarly,

$$v=(r'-b'y/r')+\frac{1}{2r'}(a'/r'^2-1/\rho)a'y^2(1+b'y/r'^2+\cdots)+\cdots;$$

therefore,

$$u+v=(r+r')-\left(\frac{b}{r}+\frac{b'}{r'}\right)y+\left[\frac{a}{r}\left(\frac{a}{r^2}-\frac{1}{\rho}\right)+\frac{a'}{r'}\left(\frac{a'}{r'^2}-\frac{1}{\rho}\right)\right]\frac{y^2}{2}+\left[\frac{ab}{r^3}\left(\frac{a}{r^2}-\frac{1}{\rho}\right)+\frac{a'b'}{r'^3}\left(\frac{a'}{r'^2}-\frac{1}{\rho}\right)\right]\frac{y^3}{2}+\cdots.$$

The contribution of each ruling to the amplitude at A' will be in phase with the contributions from all other rulings if the condition that $u+v=r+r'\pm m\lambda y/\sigma$ is satisfied for each value of y that defines a ruling. Good definition of an image of the slit at A' requires that the maximum departure from this condition be less than $\lambda/4$.

The condition is satisfied to the first order of y if A' is in such a direction that $b/r+b'/r'=\pm m\lambda/\sigma$. This is the so-called "grating condition," defining θ for each wave-length and order:

$$\sin\theta+\sin\theta'=\pm m\lambda/\sigma.$$

The condition is satisfied to the second order of y if

$$(a/r)(a/r^2-1/\rho)+(a'/r')(a'/r'^2-1/\rho)=0.$$

Restated,

$$\cos^2\theta/r+\cos^2\theta'/r'=(\cos\theta+\cos\theta')/\rho.$$

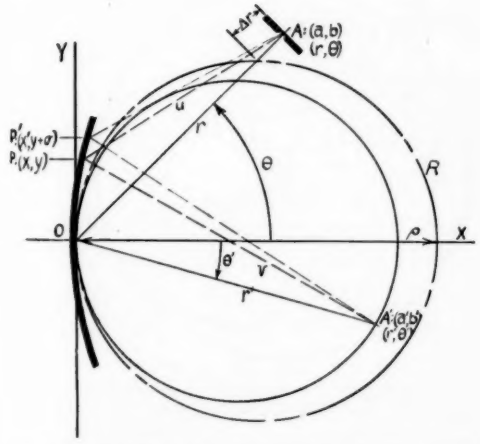

FIG. 1. Section of grating spectrograph, perpendicular to direction of rulings. P and P' are successive rulings on the surface of the grating, 0 is the point of tangency with the grating of the axis along which the constant grating interval, σ, is measured. A, the slit, and A', the point at which the diffracted light is observed, are in general at arbitrary positions in this plane of the section shown. For final focus, A and A' are made to lie on the Rowland circle, K, which is tangent to the grating at 0 and has the radius of curvature, ρ, of the grating as its diameter.

In the first case, the "grating condition" defined the direction in which light of wave-length λ would be concentrated. In the second case, the "focus condition" defines the distance from the grating at which the light in any direction from the grating is focussed. It is evident that the two degrees of freedom involved in the focussing process provide automatically for the elimination of terms in y and y^2 in the expression for the error in path length. Only one additional independent degree of freedom remains available in the spectrograph. This is the distance of the slit from the grating. Rowland showed that this sufficed to eliminate third degree errors in path length.

The condition is satisfied to the third order if $a/r^2-1/\rho=0$ and $a'/r'^2-1/\rho=0$, simultaneously, which is the case when the slit is on the Rowland circle: $r=\rho\cos\theta$. Consequently, by the focus condition, all images are also on the Rowland circle: $r'=\rho\cos\theta'$.

Differentiation of the "focus condition" shows that for moderate departures, Δr, of the slit

from the Rowland circle, all the r coordinates of the points on the focal curve differ from the corresponding r coordinates of the Rowland circle by approximately $\Delta r' = -\Delta r$.

$$(\cos^2\theta/r^2)\Delta r + (\cos^2\theta'/r')\Delta r'^2 = 0.$$

In the neighborhood of the Rowland circle, the most important case, the following relations are approximately true:

$$\cos^2\theta/r^2 = \cos^2\theta'/r'^2 = 1/\rho^2.$$

Therefore: $\Delta r' = -\Delta r$ to the same degree of approximation.[3]

We conclude, (1) that the distorted focal curve is not a circle; (2) that it approximates a circle of diameter $\rho - \Delta r$ for θ not near 90°, and $|\Delta r| < 0.05$; (3) that it is symmetrical about the grating normal which is perpendicular to the axis along which σ is measured; (4) that it does not pass through the slit ($r - r' = 2\Delta r$ when $\theta' = \theta$); (5) that it is free from aberrations of first and second degrees in y. Finally, transforming the third degree term, making use of the abbreviations,

$$a/r^2 - 1/\rho = (\rho\cos\theta - r)/r\rho = \Delta r/r\rho,$$

$$a'/r'^2 - 1/\rho = \Delta r'/r'\rho = -\Delta r/r'\rho,$$

we find

$$\Delta_3(u+v) = \left[\frac{ab}{r^3}(a/r^2 - 1/\rho) + \frac{a'b'}{r'^3}(a'/r'^2 - 1/\rho)\right]\frac{y^3}{2}$$

$$= \left(\frac{ab}{r^4\rho} - \frac{a'b'}{r'^4\rho}\right)\Delta r\frac{y^3}{2}$$

$$= \left(\frac{\sin\theta\cos\theta}{r^2\rho} - \frac{\sin\theta'\cos\theta'}{r'^2\rho}\right)\Delta r\frac{y^3}{2}$$

$$= (\tan\theta - \tan\theta')\frac{\Delta r}{2}\left(\frac{y}{\rho}\right)^3.$$

The definition of the image will remain nearly perfect until the error in the path lengths of the extreme rays becomes nearly $\lambda/4$. If the extreme rulings are at $y = \pm Y$, $|\Delta r(\tan\theta - \tan\theta')| < (\lambda/4)/(Y/\rho)^3$ assures that the definition is satisfactory. Table I shows the greatest Δr consistent with full resolving power at 5000A.

[3] This result is mentioned with others in the discussion of the Rowland mounting in Kayser, *Handbuch der Spectroscopie*, Vol. 1, 443, p. 468. The discussion of various slit adjustments in 448, p. 476 is quite complete, and applicable to most mounts.

TABLE I. *Greatest permissible departures, Δr, of slit from Rowland circle.*

$2Y$ = width of grating. ρ = radius of curvature. Values given for normal incidence, with desired images at angle θ' from normal. If slit is at angle θ from normal: $\Delta r = \Delta r_\theta \Delta r_{\theta'}/(\Delta r_\theta \pm \Delta r_{\theta'})$ where $(-)$ sign is used if θ and θ' are on opposite sides of normal. λ is assumed 5000A. Permissible Δr is proportional to λ. 5/400 column is approximately correct for 21-foot grating, 6 inches wide.

θ' (or θ)	5/100	5/200	Y/ρ = 5/300	5/400	5/500	5/600
10°	0.56 cm	4.48 cm	15.12 cm	36.0 cm	70.0 cm	121.0 cm
30	0.18	1.40	4.86	11.6	22.6	39.0
50	0.08	0.68	2.26	5.4	10.4	18.2
70	0.04	0.28	0.96	2.3	4.4	7.8
80	0.02	0.14	0.48	1.2	2.3	3.8
85	0.01	0.06	0.22	0.5	1.0	1.7
88	0.00	0.02	0.10	0.3	0.5	0.9

The conveniently simple expression for the behavior of the focal curve made possible the

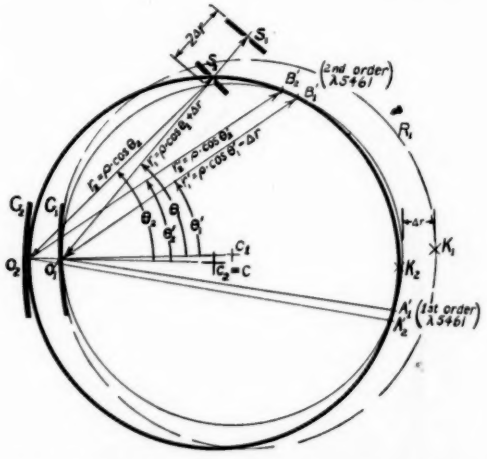

FIG. 2. Exaggerated diagram illustrating method of placing focal curve coincident with a previously established circle of diameter ρ. S_1, G_1, K_1 are preliminary positions of the slit, grating, and grating center-of-curvature. Images of slit by λ5461 for this arrangement are indicated by A_1' and B_1'. C_1 is the center of the circle through O_1, A_1', and B_1'. $2\Delta r$ is the departure of S_1 from this circle. S_2, G_2, K_2 are the corrected positions of the slit, grating, and grating center-of-curvature. C_2, the center of the circle computed through O_2, A_2', and B_2', coincides with C, the center of the established circle. This focal circle passes through the slit, S_2. Therefore the Rowland circle of G_2 is the focal curve, and coincides with the established circle (drawn heavily in the figure).

procedure indicated on Fig. 2 and accompanying legend, which placed the focal circle within the range of the plateholders. The focal positions of the first and second order green images were rapidly located by the visual knife edge test, and were brought to the center of the plate adjustment range. The various other lines were either beyond or at the extreme limits of the focus range, because of the excess curvature of the focal curve. A white paper marker was placed in coincidence with the grating center. Triangulation from two transit stations within the circle determined the coordinates of the two green images of the slit, the grating center and the slit itself. Computation of the coordinates of the center of the circle passing through the grating center and the two images, and of the radius of that circle, revealed that the slit was 4.0 centimeters outside of this circle in the direction of r. Suitable motions of the slit and grating were computed from the geometry of the arrangement, so that the slit would be brought half way to the observed focal curve and so that the new focal curve, two centimeters greater in diameter than the previous one, would again pass through the former focal positions of the green images. When these changes were completed and the grating rotated so as to bring the green images back on the plate, it was found that all the images were well within the focal range. A second determination of a circle through the green images and the grating center revealed that the slit was within 3 mm of the new focal curve, which therefore departs from the Rowland circle by less than three millimeters.

An interesting extension of Rowland's computation of the maximum useful width of a concave grating will indicate the order of agreement between the older, and simpler, criterion of resolution and the generalized criterion introduced by Mack, Stehn and Edlén for the discussion of the concave grating at grazing incidence. Restricting the discussion to the Rowland circle, the second term in the expressions for u^2 and v^2 vanishes. Then by neglecting fifth and higher degree terms in y

$$u=r-by/r+(x^2/2r)(1-a/\rho),$$

$$u+v=r+r'-(\sin\theta+\sin\theta')y+(x^2/2)[(1-\cos^2\theta)/r+(1-\cos^2\theta')/r'].$$

Therefore the lowest order error in path length is the fourth in y. By using the fourth order approximation, $x^2=y^4/4\rho^2$,

$$\Delta_4(u+v)=\frac{y^4}{8\rho^3}\left[\frac{\sin^2\theta}{\cos\theta}+\frac{\sin^2\theta'}{\cos\theta'}\right]=\frac{y^4}{8\rho^3}[\tan\theta\sin\theta+\tan\theta'\sin\theta'].$$

If we adopt Rayleigh's criterion that the definition will not be effected seriously until the extreme of the path difference of the rays from the several sections of the grating is $\lambda/4$, we find for the maximum useful width:

$$2Y_m=2\left[\frac{2\lambda\rho^3}{\tan\theta\sin\theta+\tan\theta'\sin\theta'}\right]^{\frac{1}{4}}.$$

This compares[4] with the expression found by application of the new criterion:

$$2Y_{\text{opt.}}=2.36\beta$$

$$=2.36\left[\frac{2\lambda\rho^3(2/\pi)}{\tan\theta\sin\theta+\tan\theta'\sin\theta'}\right]^{\frac{1}{4}}=1.06(2Y_m).$$

Although the new calculation is timely and redirects attention to the optimum limit of grating widths, and is a beautiful extension of Cornu's fertile method of handling problems in diffraction, it may be questioned whether the magnitude of contrast of the above results justifies the introduction of the generalized criterion of resolution.

[4] This comparison is also embodied in reference 8 of Mack, Stehn and Edlén's paper. The limiting useful values of H (directly proportional to grating width) as calculated by Rayleigh's criterion and by the new generalization of that criterion are shown to be 1.120 and 1.177, which bear the ratio of 1 : 1.053.

An Analytic Color-Tone Chart

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(Received December 1, 1932)

THE usual color chart found in books on the subject represents the various color tones of any given hue in their different degrees of brightness (value) and saturation (chroma). These tones are usually represented in a two dimensional array which may be triangular, rectangular, semicircular or irregular in outline, and which in turn may represent the section of some solid figure embracing similar tones for all hues. Present-day schemes for color representation are for the most part lineal descendants of the pyramid of Lambert, the sphere of Runge and the cylinder of Chevreul. The well-known Munsell[1] system with its convenient notation is based upon the sphere but the actual shape of the figure has been modified considerably, while the schemes of Ross[2] and Pope[3] of Harvard incline toward one or other of two triangular arrangements for the tones of any given hue.

Any color solid represents an attempt to put the points of some three dimensional figure into correspondence with color sensations,[4] and the shape and limits of such a solid must depend upon the scales of reference which are chosen and the limitations which are imposed. Such schemes as those mentioned above do not obviously represent the proportions of the components of each tone but suggest them only, whereas the chart herewith presented, Fig. 1, shows each tone analyzed into its physical components for an ideal case the resultants of which can easily be approximated with the color disk. In thus displaying graphically the relation of each tone to every other, the chart may serve three general purposes; first, to form a simple ideal arrangement with which actual measurements may be easily compared; second, to offer a convenient means of evaluation of the present varied and confused terminology in use by both artists and scientists; and third, to be useful for purposes of instruction.

The vertical scale of brightness has been arbitrarily divided into nine uniform divisions, and each tone represented by a large square has been arbitrarily divided into sixteen units for convenience. Ascending the vertical column AB and assuming the blacks are perfect blacks it is seen that the brightness increases by steps of two white units each. Along any horizontal row the color tone changes by similar uniform steps from neutral gray on the left to the most saturated value the tone may have at that brightness level. The introduction of such equal steps between the three extremes, white (A), black (B), and red (C) leads directly to the triangular arrangement shown in Fig. 1 as giving the simplest relation between the various tones.

The number of red units to be exchanged for two white units in order to keep the brightness the same should be predetermined by photometric measurement for any real case but for purposes of an ideal chart it is here assumed (i.e., the red is so chosen) that four unit squares of red are equivalent in brightness to two unit squares of white. The apex of the triangle is therefore at C half-way on the vertical scale. In terms of colored lights this means that the maximum brightness of the reflected red is one-half the maximum brightness of the reflected white. For a red of different brightness or for another hue, the apex of the triangle would move up or down, fractions would be involved and the chart would need appropriate revision, however its outline will remain triangular. The division of the rectangle representing each color

[1] A. H. Munsell, *A Color Notation*, Munsell Color Company, 1923.

[2] D. W. Ross, *A Theory of Pure Design*, Houghton, Mifflin & Company, 1907.

[3] A. Pope, *The Painter's Terms*, Harvard University Press, 1929, p. 21.

[4] L. T. Troland, *Psychophysiology*, Van Nostrand, 1929, p. 250.

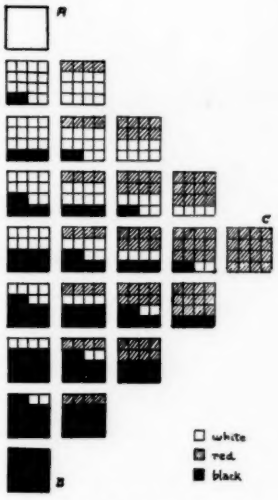

FIG. 1. An analytic color-tone chart.

tone into a hundred units would of course reduce the proportional relations to a simple percentage basis.

The primary problem for the artist is to be able to describe mixtures of pigments, whereas the physicist may be as much or more interested in describing combinations of colored illuminants. The chart may be interpreted in terms of either of these processes or in terms of relative areas of paper for the color disk. In the latter case the reflectivity of the black, and in the first case the chemical and physical effects of mixing pigments, will, of course, produce deviations of the real from the ideal. By using the proportions of the chart as a basis, the tones produced by colored illuminants or by the color disk may be compared with each other, or either may be compared with actual pigment tones. Deviations of the real from the ideal may thus be determined.

In the confusion of terminology[5] which yet exists between scientists and artists regarding color tones, many different names have been applied to the scales represented not only by the horizontal and vertical rows but by the diagonals in the AC and BC directions and even certain sectors of the angles at A and B. The present chart makes it a much more simple matter to compare and evaluate these terms. Furthermore, arrangements of tones other than the present triangular arrangement may likewise be evaluated by a similar analysis.

Take for example the simple cases of the terms "tint" and "shade." If we say that "a tint is a pure color mixed with white," then, if the chart represents reflected colored lights, all the tones are tints except along BC. If the chart represent pigments then only the tones on the diagonal AC are pure tints though all of the tones in the upper half of the chart are often so called. All other tones in the upper half of the chart contain an additional factor of gray, and may be called impure tints. By the same standard the tones along BC represent, in terms of pigment, shades or mixtures of the pure color with black. In terms of colored lights, they represent merely different fractions of the maximum intensity at C of the pure color. The chart makes evident the variation of meaning because of the difference in viewpoint.

Although in passing from one vertical column to the next one on the right the equal exchange of two white units is always made for four red units, full saturation at each level is gained in different numbers of equal steps. If an arrangement other than triangular is used and the numbers of steps equalized, then the steps themselves become unequal. This is doubtless one of the causes of the suggestion that a hyperspace may be needed for a complete description of color-tones, but it can scarcely be thought of as a simplifying assumption and the chart explains away some of these difficulties by making them obvious.

[5] A. Pope, reference 3, pp. 49–62.

On the Concave Grating Spectrograph, Especially at Large Angles of Incidence: Addendum

J. E. MACK AND J. R. STEHN, *Department of Physics, University of Wisconsin*

(Received February 6, 1933)

A graph is given from which the optimum width of a concave grating may be found for any practical case.

IN a recent paper with this title[1] an expression was derived for the optimum width W (formerly called $2Y_{opt}$) of a concave grating as a function of the wave-length λ, the angle of incidence θ or of excidence θ', the grating radius ρ, the grating interval σ, and the spectral order m. The purpose of this addition is to present a graph (Fig. 7) from which one may calculate W fairly accurately for all practical cases.

Using the grating condition equation, $\sin\theta' = m\lambda/\sigma - \sin\theta$, we have

$$W=\left[\frac{39.5\lambda\rho^3}{\sin\theta\tan\theta+(m\lambda/\sigma-\sin\theta)\tan\arcsin(m\lambda/\sigma-\sin\theta)}\right]^{\frac{1}{4}}. \tag{13}$$

Fig. 7 shows W as a function of λ, for the values of θ indicated. On account of the dependence of W upon m/σ, a family of curves ought to be drawn for each θ, but instead, for simplicity, the practical range of W for each θ is shown by an unshaded area. This range can be seen from equation (13) to have its upper boundary at $m\lambda/\sigma=\sin\theta$ (i.e., at $\theta'=0$); its lower boundary is arbitrarily chosen as the curve $m/\sigma=2000$ cm^{-1}. (Thus "inside orders," the spectra spread from the central image toward the grating, are omitted.) Sample curves are drawn for the first, second, third, and twentieth orders from a 12,000 line/cm grating, for $\theta=45°$ and $\theta=89°$. These would drop to $W=0$ for $\lambda=(1+\sin\theta)\sigma/m$, but in the figure they are cut off where the ray retraces its path (at $\lambda=2\sin\theta\cdot\sigma/m$, beyond the practical range). In addition, the corresponding curves are shown for $\theta=0$; because of the symmetry of W in θ and θ', each of the latter curves crosses the curve for another θ and the same m/σ at the top of the practical range for that θ.

The main body of the figure shows ordinates corresponding to a one meter grating ($\rho=100$ cm). For any value of ρ the graph may be used after multiplication of the main ordinate scale by $(\rho/100\text{ cm})^{\frac{1}{4}}$; scales for certain commonly used radii are given at the left. When the optimum width is known, the resolving power of any grating may be found by reference to Fig. 4.

[1] J. E. Mack, J. R. Stehn and B. Edlén, J. O. S. A. **22**, 245 (1932).

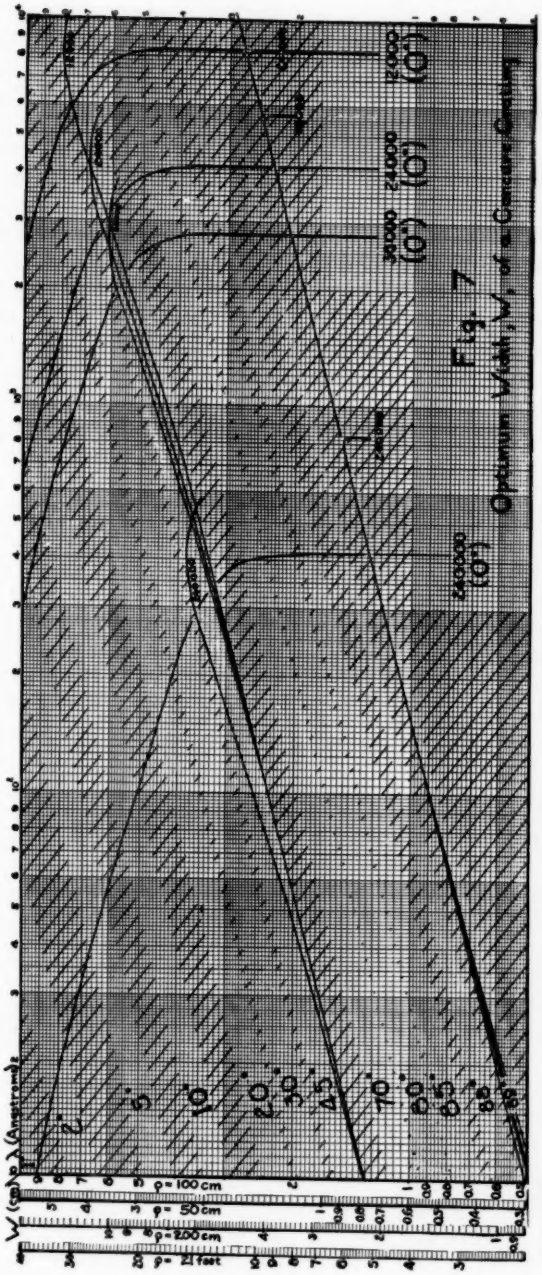
Fig. 7
Optimum Width, W, of a Concave Grating
30000 (0°)
24000 (0°)
18000 (0°)
24000 (0°)
ρ = 100 cm
ρ = 50 cm
ρ = 250 cm
ρ = 21 feet
W (cm)
λ (Angstroms)

FIG. 7.

A New Type of Spectrographic Slit

G. FASSIN, *Institute of Optics, University of Rochester*

(Received January 27, 1933)

ONE of the most difficult problems in the design of a spectroscope is undoubtedly the bilateral slit mechanism.

Some of the most important requirements are: (1) That the slit should open with its jaw edges strictly parallel. (2) That it should close by spring tension. (3) The effort to close the jaw blades should be minimized so that this spring tension also can be reduced to a minimum. (4) The mechanism of opening and closing should be extremely sensitive.

A great number of devices more or less complicated have been supplied without real success. A good slit is generally expensive as it requires skilled and specialized labor and the greatest precision in every mechanical part. Attempting to meet all these requirements in the simplest and most successful way, we built the geometrical slit shown in Fig. 1. Two U-shaped jaw carriers rotate around a main axis fixed in the slit support. The jaw carriers are held together by means of springs and they are wedged open by a cone at the end of a micrometer screw. The slit support S after being machined was fixed on to a surface plate with 90° angle support and the hole H was drilled and reamed so that both steel pins P_1 and P_2 were accurately centered. C_1 and C_2 are the slit jaw carriers; both of these carriers were machined together so the distance from the V grooves V_1 and V_2 to the surface S_1 was very precisely the same in both pieces. C_1 and C_2 are mounted on to the pins P_1 and P_2 and kept in contact by means of two springs L_1 and L_2. The ends of these springs are set in two little cavities drilled into C_1 and C_2 a little below the center line of the V grooves, and the middle of the spring fits in a small groove milled into C_1 and C_2 to keep them in place. The great length of the spring favors of the sensitivity of the slit mechanism.

Screw M (1 mm) (Fig. 2) mounted in a spring nut, is tapered on the lower end and wedges

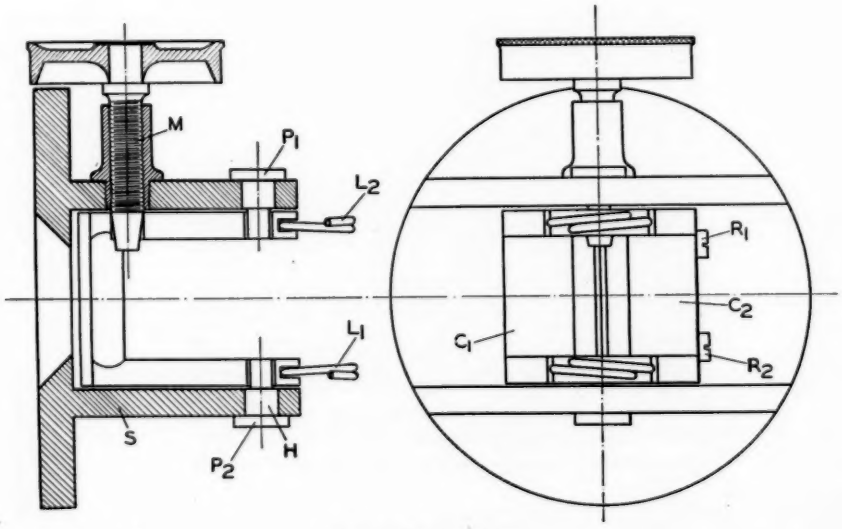

FIG. 1. Slit assembly.

between the two jaw blade carriers C_1 and C_2 in which two inclined planes are provided. The taper of the screw should be made perfectly concentric with the screw thread. If the cone is turned at the same time the thread is cut, this operation presents no difficulties. The jaw blades were made of Stellite and optically lapped by a method suggested by Mr. A. Callier.[1] It is known that a good slit jaw should have a very sharp edge. However, if we finish the jaw on the angle as shown in Baly's *Spectroscopy*, Vol. I, Chapter III, Fig. 18, this edge becomes extremely fragile. In using the Callier scheme we can easily correct for this. The jaw blades are clamped between two holders on an angle of 45°.

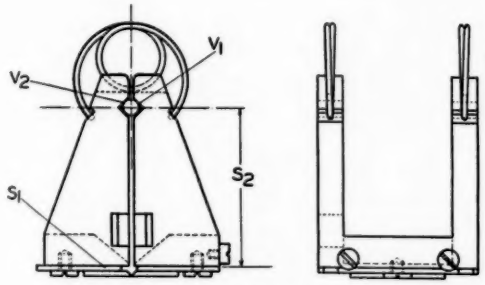

FIG. 2. Jaw blade carriers.

First we lap off on an optical flat side A so that the slit blade presents a knife edge under a 45° angle; then we turn the mount over 90° and bevel off the other edge very slightly so as to create a 90° angle and reinforce the sharp edge considerably. The slit blade can be finished entirely before removal from this mount. (Fig. 3.)

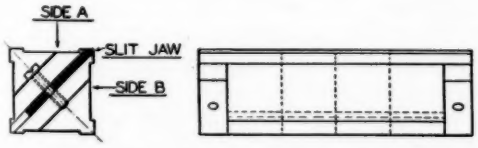

FIG. 3. Slit jaws; first and second lapping.

One blade is fixed on to the carrier C_1 with one screw in the middle; two screws R_1 and R_2 will adjust the parallelism between the other blade which is fixed with these screws on to carrier C_2. (Fig. 4.)

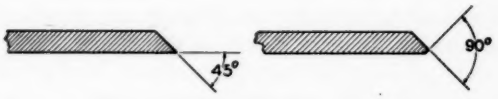

FIG. 4. Device for grinding slit jaws (as suggested by A. Callier).

POSSIBLE ERRORS

In considering that the slit rotates around a point we see immediately that we introduce an error in the mechanical distance, from slit to prism of the spectrograph, which is equal to

$$E = D - (D \cos \alpha),$$

D being the distance from the slit edge to the center of rotation and angle α the angular opening, of the slit, from the central axis.

However, we will find that even in spectrophotometric work we never have to open the slit more than 1 mm so that this error E is approximately 2 microns or an imperceptible distance considering the depth of focus of the spectrograph. If the two center pins even are out of line by 3/100 mm (which is a large tolerance for instrument work) we will find that the error introduced is infinitesimally small. The possibility of eccentricity between the cone and the micro-screw for opening the slit is more serious, because that would produce a change in line position with respect to the wave-length or millimeter scale printed on the plate.

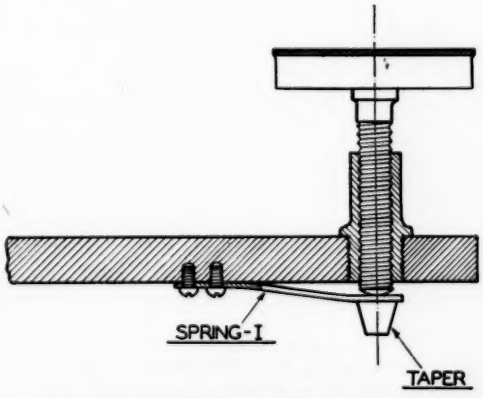

FIG. 5. Screw arrangement with separate wedge.

[1] A. Callier, Director of the Belgian Optical Society, Ghent.

As already mentioned, this error can be avoided in construction. If the tolerance of this error is extremely small, however, or if the mechanism is to be constructed by unskilled mechanics, this error may be eliminated with a slight change in design, as shown in Fig. 5. In this case the cone or wedge would be completely independent of the rotation of the screw. The spring (*I*) would always assure contact between screw and wedge.

This slit mechanism was entirely built by the students in the course in mechanical design at the Institute of Optics and worked very successfully. This proves that neither specialized labor nor high precision are required to produce a very satisfactory spectrographic slit.

Proceedings of the New York Meeting of the Optical Society of America

FEBRUARY 24–25, 1933

JOINT MEETING WITH THE AMERICAN PHYSICAL SOCIETY

The joint meeting of the Optical Society of America and the American Physical Society was held in New York City, February 24 and 25, 1933.

JOINT SYMPOSIUM ON ELECTRON OPTICS

The principal feature of the meeting was the joint symposium of the two societies on Friday afternoon, February 24, devoted to invited papers on *electron optics*. The abstracts of these papers are appended.

JOINT DINNER

President Crittenden of the Optical Society presided at the joint dinner, Friday evening, and announced the completion of the incorporation of the Optical Society.

President Foote of the American Physical Society in a brief talk called attention to the close relations existing between the two societies and Professor Urey outlined the plans of the new *Journal of Chemical Physics*.

CONTRIBUTED PAPERS

Simultaneous meetings of the American Physical Society and the Optical Society of America for the reading of contributed papers were held on Friday morning and Saturday morning. The authors' abstracts of the papers presented before the Optical Society are appended.

EXECUTIVE COUNCIL REORGANIZED AS BOARD OF DIRECTORS

The Executive Council met Thursday evening at 8.00 P.M. and reorganized as the Board of Directors of the Optical Society of America, Incorporated.

L. B. TUCKERMAN, *Secretary*

Minutes of the First Meeting of the Directors of the Optical Society of America, Incorporated

The meeting was called to order at 8.00 P.M. Mr. E. C. Crittenden was chosen temporary chairman and L. B. Tuckerman, temporary secretary.

The Chairman reported that the certificate of incorporation of the Optical Society of America, Incorporated had been duly filed in the office of the Secretary of State at Albany, New York, on October 26, 1932.

It was then voted that the present officers of the Optical Society of America, Unincorporated, to wit:

E. C. Crittenden, *President*
W. B. Rayton, *Vice-President*
L. B. Tuckerman, *Secretary*
H. F. Kurtz, *Treasurer*

be elected officers of the Optical Society of America, Incorporated, to serve until the election of their successors, in accordance with the by-laws.

The Directors then took up the regular order of business.

Besides the transaction of routine business the following items of general interest to members of the Society were taken up.

The Secretary reported the following appointments made since the last meeting of the Executive Council of the Optical Society of America, Unincorporated:

Appointment by Special Committee:

H. F. Kurtz, Treasurer, to serve until the next regular election.

Appointments by the President:

Auditors of the Treasurer's Accounts
Carl L. Bausch
C. W. Frederick

Auditors of the Accounts of the Journals:

G. E. Grantham
Frederick R. Hirsh, Jr.

Appointment by President and Secretary:

Proxies at the Annual Meeting of the American Institute of Physics, February 24, 1933:
Loyd A. Jones
F. K. Richtmyer

Appointments Approved by Letter Ballot:

Members of the Colorimetry Committee:
Loyd A. Jones, Chairman (previously appointed)
Elliot Q. Adams
Charles Bittinger
H. Phelps Gage
K. S. Gibson
I. H. Godlove
Arthur C. Hardy
Herbert E. Ives
Deane B. Judd
Earl M. Lowry
A. E. O. Munsell
Dorothy Nickerson
M. Rea Paul
W. B. Van Arsdel

Appointments by the President Approved by the Directors:

Nominating Committee for the Regular Election to be Held in October:
F. K. Richtmyer, *Chairman*
I. C. Gardner
George R. Harrison
Brian O'Brien
Harold W. Webb

Ives Medal Committee for 1933
Karl K. Darrow, *Chairman*
J. A. Anderson
Karl T. Compton
Loyd A. Jones
H. M. Randall

The Special Committee on Publications reported that in accordance with the authorization of the Council the contract with the American Institute of Physics for the publication of the Journal during the year 1933 had been signed.

The Committee on Incorporation and Revision of the By-laws presented a final revision which was approved and ordered printed in the Journal and submitted to vote of the regular and honorary members of the Society.

At the request of the treasurer the Directors voted that he secure a bond in the sum of $7000.00 and that the Security Trust Company of Rochester be made fiscal agent for endowment or permanent funds of the Society, subject to the instructions of the president and treasurer or vice-president and secretary.

Arrangements for the 1933 election were completed (as announced in the Journal of the Optical Society of America **23**, 122 (1933).

L. B. TUCKERMAN, *Secretary*

Symposium on Electron Optics

JOINT SESSION OF THE AMERICAN PHYSICAL SOCIETY AND THE OPTICAL SOCIETY OF AMERICA

(INVITED PAPERS)

1. Electron Optics

C. J. DAVISSON
Bell Telephone Laboratories, Inc.

The term "electron optics" is used to designate the theory and practice of focussing beams of electrons by electric and magnetic fields. Interest in this subject—by no means a new one—has increased greatly during the last few years. In 1926 Brusch showed by calculation and demonstrated by experiment that the field of a short circular coil magnet has for electrons of a given speed the properties of a positive lens of definite and calculable focal length. The possibilities of electron optical systems in which this type of lens is employed have been explored recently by Ruska and Knoll. Sharply defined images of stencils have been produced and magnifications up to 400 times have been obtained. Images of emitting surfaces also have been formed by these experimenters. The magnetic lens systems is not so well adapted to projection of this kind, however, as the purely electrical system. In this the "lenses" are the distorted fields about apertures in charged electrodes. Brüche and Johannson have demonstrated the important possibilities of this type of system in portraying the "brightness" patterns of emitting surfaces. Our interest in this subject in the Bell Telephone Laboratories has been primarily in the production of beams of electrons of prescribed characteristics—in improvement of the so-called "electron gun" as a collimator. A theory of electron optical

systems of the electrical type has been developed. Its usefulness for purposes of electron optical design has been demonstrated in numerous tests.

2. Diffraction of Low-Speed Electrons

H. E. FARNSWORTH
Brown University

Experiments with low-speed electrons have shown many deviations from the results to be expected on the basis of de Broglie's original relation. These deviations exist for normal incidence as well as for glancing angles of incidence used in the Bragg method, and may appear as a secondary or fine structure of the main diffraction beams. The relative intensities of the various components are extremely sensitive to small changes in the angle of incidence. This results in large variations of the relative integrated intensities of the main diffraction beams with small changes in angle of incidence. It thus appears that an understanding of the phenomenon should contain information on still obscure points in the present view of the solid state, such as the concept of an average inner potential, deviations from such an average value, and interaction between electrons in crystals. The following further experiments have been undertaken to determine possible surface effects, and in particular the "surface action" considered by v. Laue. (1) A study of the characteristics of a thin film of one metal deposited by evaporation on a single crystal of another metal. (2) Investigation of reflection from opposite sides of a given set of atomic planes when using primary electrons which enter different faces of the same crystal in the two cases. The first experiment shows that the fine structure characteristics for the film are essentially the same as those for a massive crystal of the same metal, and thus eliminates surface irregularities as a possible cause of fine structure differences. The second experiment indicates that there is an unsymmetrical reflection from opposite sides of an atomic plane, and appears to have a significant bearing on the interpretation of the fine structure.

3. A Lens for Cosmic-Ray Electrons

W. F. G. SWANN AND W. E. DANFORTH, JR.
Bartol Research Foundation of the Franklin Institute

The design involves the use of two vertical concentric tubes between which a potential difference is maintained. Along the axis of the tube two counters are placed, one above and the other below. It is then possible to focus the rays passing through one counter so as to pass through the other. The main features of the apparatus lie in the design of the tube. The fact that the field decreases with the distance from the axis, whereas in order to produce focussing it would have to increase, necessitates a compensation produced by causing the rays which are required to be bent through the greater angle to traverse a longer path than the others. The desired result is secured by flaring the ends of the tubes in such a manner that the different rays enter the field and also leave it at different points. The portions of the tube between the end flares have to be properly formed in order that the rays shall not be lost by passing through the walls. The characteristics may be described as follows. The tube system is one meter long. The distance between the two tubes is of the order of one centimeter, but varies along the length. The mean radius is five centimeters, at the entrance end. 10^8 volt-electrons passing through a point one meter above the upper end of the tubes would be focussed to a point one meter below the lower end, by a potential difference of 100,000 volts between the two cylinders. A ring counter is placed at the entrance end of the tubes, so that the rays which are recorded are those which pass through the ring counter and the conjugate foci. With the conditions cited, about five counts per day would be expected if all of the corpuscular rays had 10^8 volts velocity. A test of the two counters together with the circular counter in the absence of the cylinders should of course reveal a number of accidental counts sufficiently small to avoid masking the main effect. A preliminary test has indicated only one count in thirty-six hours with this arrangement. If the linear dimensions of the apparatus be increased certain advantages are gained. The field necessary to produce focussing of rays of assigned energy diminishes inversely in proportion to the linear dimensions, so that with an apparatus extending over a length of thirty meters, a field strength of 15,000 volts per centimeter may be utilized and the apparatus can be used in air. The principal advantage, however, is gained in an increase in the number of rays which can be obtained. Increase of the linear dimension in a constant ratio does not increase the solid angle but it does increase the area permissible for the initial counter. With a system thirty meters long, the counts can be increased to such an extent as will permit 500 counts per day and provide a source of radiation of convenient strength for use in experiments. By increasing the maximum field between the tubes to 20,000 volts per centimeter, which is permissible, the distance between the counters may be shortened to about 20 meters, which is a distance attainable without very much difficulty. This would increase the number of counts.

4. On Electron Beams in High Vacuum

VLADIMIR K. ZWORYKIN
Research Division, RCA Victor Company, Camden, N. J.

In order to concentrate a beam of electrons into a high intensity, sharply defined bundle, various methods have been used. The concentration of electron beams in high vacuum depends entirely upon fields of force, either electric or magnetic, while in low vacuum or in rarefied rare gas it depends also upon the action of gas molecules. In high vacuum the fields of force act upon the electron beam similar to the action of lenses upon a beam of light. An improperly shaped field produces effects similar to spherical aberration in poorly corrected lenses; non-uniform velocity of electrons in the beam results in effects similar to chromatic aberration of light. This optical analogy is not perfect. In the case of the electron beam the velocity varies continuously throughout most of the path and indices of refraction employed usually are greater than in the optical

case. Moreover, space charge in beams of high intensity limits the concentration attainable: consequently, even theoretically, an electron beam can never be brought to a mathematical point as in the case of light optics. Other secondary effects are described. In focussing electron beams, both electrostatic and electromagnetic methods have been used extensively. The electrostatic method, however, seems to be preferable, especially when the beam is to be deflected. Precautions should be taken not to destroy the focussing of the beam during deflection. A list of references on the subject is given at the end of the paper.

Program of Sessions for Contributed Papers

(TITLES AND ABSTRACTS* OF PAPERS)

5. Photographic Plates for Use in Spectroscopy and Astronomy III. C. E. KENNETH MEES, *Eastman Kodak Company.*

As a result of experimental work and of experience with users during the past year, a number of modifications have been made in the photographic materials described in previous communications.

A new emulsion is described of very low speed and very fine grain and high resolving power, suitable for spectrographs of low dispersion and high intensity.

The various sensitizings for different portions of the visible spectrum have been modified somewhat and a new sensitizing showing a high maximum at λ6850A is described.

High-speed emulsions have now been sensitized for the infrared. Very good sensitiveness is obtainable by a series of four plates covering the region from λ6800A to λ10,400A with maxima at λ7200A, λ8200A, λ8600A, and λ9600A. The exposures through this region of the infrared are now less than one-third of those obtainable upon the materials previously described.

The most marked success has been attained with the new sensitizing for the regions of the infrared close to the visible spectrum.

The sensitizing with xenocyanine has been improved, and with this dye reasonably short exposures can be obtained out to λ12,000A.

"Photographic Plates for Use in Spectroscopy and Astronomy, I" Journal of the Optical Society of America 21, 753 (1931); Part II, *ibid.* 204 (1932).

The present paper will appear in full in Journal of the Optical Society of America.

6. The Intermittency Effect and the Reciprocity Law Failure in Photographic Exposure. J. H. WEBB, *Kodak Research Laboratories.*

Experimental determinations of the intermittency effect and the reciprocity law failure were made on the three types of emulsions: high-speed ordinary, low-speed ordinary, and high-speed panchromatic. It was found that the intermittency effect became identical with the reciprocity law failure for frequencies of flash above a critical value, f_c, which varied with different emulsions and with the average intensity I_{av} at which the intermittent exposure was made.

A theory based on the assumption that f_c is that frequency which allows on the average but one quantum per flash per effective receptive area ρ of a grain, yields the formula $f_c = \rho I_{av}$. This formula agrees with the experimental facts found and further predicts the size of the effective receptive area ρ of the photographic grain. Values of ρ calculated from the above formula agree in order of magnitude with the size of the photographic grain, indicating that the entire grain is the effective receptor of quanta in exposure.

The present paper will appear in full in Journal of the Optical Society of America.

7. The Photometry of Solar Eclipse Phenomena. CLAYTON H. SHARP, S. MCK. GRAY, W. F. LITTLE, H. J. ECKWEILER, *Electrical Testing Laboratories.*

The Electrical Testing Laboratories of New York desiring to supplement the work in the photometry of eclipse phenomena which it had done through three expeditions at the time of the 1925 eclipse, designed apparatus and methods and sent an expedition to Lancaster, New Hampshire, to observe the eclipse of August, 1932. In preparing for this expedition features connected with the photometry of eclipse phenomena were considered anew and are discussed.

The use of the term "brightness" with reference to the solar corona is criticized as misleading and not in accordance with photometric usage. The quantity to be measured is the illumination produced by the corona.

Discussion is made of erroneous and misleading values for corona illumination which have been published and which refer to illumination produced not by the corona alone but by the corona together with a large area of sky and which have no especial scientific significance.

Specially designed photometers are described for measuring the coronal light, the horizontal illumination and sky brightness. These have filters to give a color match in the photometric field as closely as the color to be encountered can be predicted. They have easily manipulated neutral screens to cover the range of intensities. The coronal photometer has an optical system whereby the field included is accurately defined and a "homemade" heliostat to keep the sun in position. There is also a photoelectric photometer for corona measurement consisting of a vacuum photoelectric cell with colored filters correcting its response

* The abstracts were preprinted in the program subject to correction by the authors. Corrections that have been communicated to the secretary have been made in the abstracts here reprinted.

to the eye sensitivity curve. This acts through an amplifier with an FP-54 tube and is fitted with an optical system for delimiting the field. Special recording arrangements are described.

Cloudiness at Lancaster prevented any measurements of the corona light to be obtained. It was found that the total illumination on a horizontal plane at the time of totality was 0.35 footcandles.

8. Ultraviolet Radiation from Some Selected Sources. W. E. FORSYTHE, B. T. BARNES AND M. A. EASLEY, *Lamp Development Laboratory, General Electric Co., Nela Park, Cleveland, Ohio.*

The amount of ultraviolet radiation within a definite region and the spectral distribution within this region are important characteristics of ultraviolet sources. The spectral distribution for the radiation of wave-lengths shorter than 3130A has been measured for a number of ultraviolet sources and the values given in Table I. From the values in the table the total energy within this region may be obtained by correcting for the weaker lines and for the continuous radiation. The distribution of ultraviolet

TABLE I.

Source		Quartz mercury arc	S−1 Unit	S−2 Unit	G−1 Unit	G−5 Unit	500 watt Mazda CX
Lamp voltage		75.	14.1	16.2	19.7	13.6	110.
Lamp current		3.75	27.6	8.0	2.0	5.2	
Distance		100 cm	76.	61.	46.	61.	76.
Micro-watts per cm²	Wave-length						
	3129	70.	173.	45.	13.	13.	
	3022	31.	59.	12.	1.7	1.5	
	2967	15.6	22.0	5.4	2.2	2.2	
	2925	1.9	1.9	.37	.1	.1	
	2894	5.7	5.4	1.35	.6	.6	
	2804	10.9	4.1	.90			
	2652	23.4	.96	.35			
	2537	32.					
	2483	9.4					
Total for λ3130 and shorter			276	68	17.5	17.5	1.07

around the various units, the variation of the ultraviolet with distance from the units and the effect of tilting the units on the output have been measured with a photo-electric tube and the results will be shown in form of curves.

The present paper will appear in full in Journal of the Optical Society of America.

9. A Photo-Relay for the Amplification of Galvanometer Currents. PAUL R. GLEASON, *Colgate University.*

The surface of a Weston Photronic Cell has been cut to provide a differential photo-cell similar to that recently described by Bergmann. The image of a straight filament is focussed after reflection from a galvanometer mirror on the boundary between the two halves of the differential cell. If the image is symmetrically placed, and if the two halves of the Photronic cell are of equal sensitivity, a null reading will result in the second galvanometer which is connected in series with the two halves of the cell. A very small motion of the mirror of the first galvanometer will displace slightly the image on the differential cell and will result in an appreciable deflection of the second galvanometer.

The device resembles the thermal relay but is simpler, more flexible and requires no low-resistance galvanometer in the output. With two galvanometers, each of 2×10^{-10} amp. sensitivity, one as primary galvanometer the other measuring the output of the differential cell, it is easy to observe the random fluctuations characteristic of the upper limit of sensitivities attainable with any device, corresponding to about 10^{-12} amp. in the primary galvanometer. Over useful limits the response is practically linear. Neither magnetic or electrostatic shielding is required in our laboratory although all circuits have to be grounded. Many factors enter into the overall sensitivity of the device. The characteristics can be improved by cutting the differential cell entirely in two and connecting them electrically in parallel opposing, with the output galvanometer also in parallel.

The photo-relay is being used to measure the small currents from barrier-layer cells subjected to very feeble illumination (approximately 10^{-6} footcandles). It is also being used to study the residual currents the barrier-layer cells seem to exhibit in the dark.

Bergmann, Phys. Zeits. **33**, 824 (1932).

The present paper will appear in brief in Letters to Editor, Physical Review.

10. A New Type of Spectrographic Slit. GUSTAVE FASSIN, *Institute of Optics, University of Rochester.*

This article describes a spectrographic slit comprising a support having a main axis on which the two jaw carriers are swingably mounted. The jaws are normally urged into contact with each other by springs. The jaws are separated by a screw having a cone-shaped end which exerts a camming action on two inclined portions on the respective jaw carriers. Although the slit comprises a minimum number of parts of relatively simple construction the assembled unit affords an extremely sensitive and accurate slit.

11. A New Spark Stand with Rotatable Electrodes. GUSTAVE FASSIN, *Institute of Optics, University of Rochester.*

To maintain a constant illumination between two electrodes over long periods, two annular electrodes are rotatably mounted one above the other with their axes of rotation at right angles to each other. The annular electrodes are removably mounted on flat disk carriers which are supported on hollow insulated metal shafts. Two motors with suitable speed reducing gears serve to rotate each electrode at the rate of one revolution per hour. A blast of air is forced through the hollow shaft for the purpose of cooling the electrode. Suitable means are provided for adjusting the vertical distance between the vertically spaced electrodes.

12. A Way of Producing Gratings Giving Lyman Ghosts. D. T. WILBER AND C. W. GARTLEIN, *Cornell University.*

Lyman ghosts are explained by Runge[1] as due to a combination of an error of long period with an error of

short period. The short period has been assumed to come from an error introduced by the driving mechanism.

We have found that as the same part of the diamond rules on glass for a large number of lines, Lyman ghosts begin to appear. When examined under the microscope the diamond edge then appeared flat and crossed by grooves worn by the glass in the diamond at apparently the same spacing (15,000 lines per inch) which had been ruled on the glass.

This further is evidenced by later ruling with the same diamond 3750 lines per inch, giving a grating in which the deep grooves at that spacing have three shallow grooves between each pair at 3750 per inch. These are undoubtedly produced by the grooves on the diamond.

When ruling, the periodic error of the screw tends, once in 500 lines, to make the distance between the lines greater than the normal spacing. The lines on the diamond, meshing with the lines previously ruled on the glass would tend to keep this distance less at this position, and greater when the error of the screw tends to make the spacing less than normal.

There would thus be introduced a compound error based on the period of the screw and on the width of the diamond surface or the number of lines worn in the surface. A much worn surface may exceed ten lines in width. The auxiliary error found by Runge in gratings is three or five lines.

We have ruled gratings showing Lyman ghosts at 5/7, 6/7, etc., of the true deviation for 15,000 lines using a diamond that was much worn but not altering the machine in any other way from the condition in which it ruled spectra free from Lyman ghosts.

The present paper will appear in full in Review of Scientific Instruments.

[1] Runge, J.O.S.A. and R.S.I. 6, 429 (1922).

13. Colorimetry with Reflection Standards: A Quasi-Psychological Method and Data for the Inter-Conversion of Physical and Psychological Color Specifications. A. E. O. MUNSELL AND I. H. GODLOVE, *Munsell Research Laboratory.*

In order to test the accuracy and practicability of the calibration and use of Munsell reflection standards (a quasi-psychological method), and if found sufficient for the purpose, to forge the final connecting link in the conversion chain between an extreme physical method (spectrophotometry) and the purely psychological method of specifying colors, 32 colored papers were calibrated by two methods. One was the usual direct comparison visually with the standards; the other was an indirect method involving spectrophotometry, colorimetry with the Guild trichromatic colorimeter, and conversion to hue and chroma by means of the relations previously found for 400 Munsell colors.[1] In spite of known small errors, the overall check was good. It is therefore possible, through the mediation of these calibrated colors, to obtain at least an approximate color or color-stimulus specification (for observers sufficiently near the standard one assumed in the case of computations from spectrophotometry) in the terms of any of the usual methods, if a specification by any of the other methods is given.

The present paper will appear in full in the Journal of the Optical Society of America.

[1] Paper to be published in J. O. S. A. read by I. H. Godlove at the Cambridge (1932) meeting of the O. S. A.

14. Computation from Spectrophotometric Data of Trilinear Coordinates, Dominant Wave-Length, Colorimetric Purity and Relative Brightness on the 1931 C.I.E. Basis. DEANE B. JUDD, *Bureau of Standards.*

Forms have been drawn up for the computation of trilinear coordinates (trichromatic coefficients) and relative brightness (reflectance or transmission) of samples illuminated by any one of the following illuminants: Planck 1900, 2360 and 2848°K and Davis-Gibson lamp-and-filter equivalents of Planck 4800 and 6500°K. The three last-named illuminants are the 1931 C.I.E. standard illuminants *A*, *B* and *C*, respectively. A form has also been drawn up for the computation of dominant wave-length and colorimetric purity from two given sets of trilinear coordinates, the first set specifying the heterogeneous component in the homogeneous-heterogeneous mixture and the second set specifying the particular illuminant-sample combination whose dominant wave-length and purity are desired. The basic stimulus (stimulus represented at the center of the Maxwell triangle) for all these computation forms is the equal-energy spectrum in conformity with the 1931 C.I.E. coordinate system.

The data included on the trilinear-coordinate forms differ from those published by Smith and Guild[1] only because they refer to intervals of 10mμ instead of 5. For the great majority of practical cases the 10mμ intervals are sufficiently accurate; cases of very rapid change in distribution of energy with wave-length occur, however, which require special methods of computation because the use of even 5mμ intervals results in appreciable error.

The method of computing dominant wave-length and colorimetric purity differs from previous methods chiefly in three ways: (1) it provides for computation with respect to any heterogeneous stimulus instead of merely the basic stimulus by using a general formula for purity,[2] (2) it takes advantage of the simplification embodied in the 1931 C.I.E. coordinate system by reason of the inclusion of the standard visibility function as one of the distribution curves, and (3) it makes use of linear interpolation from a table giving values for every millimicron rather than the reading of interpolation graphs.

[1] T. Smith and J. Guild, *The C. I. E. Colorimetric Standards and Their Use*, Trans. Opt. Soc. **33**, 103–105 (1931–32).

[2] D. B. Judd, *A General Formula for the Computation of Colorimetric Purity*, J.O.S.A. **21**, 729–747 (1931).

15. The Influence of Intensity on the Critical Fusion Frequency of Intermittent Illumination for Different Retinal Locations and Colors. SELIG HECHT AND CORNELIS D. VERRIJP,* *Columbia University.*

The relation between intensity and critical fusion frequency has been studied over the full range of intensities to which the eye responds. When the measurements are confined to the rod-free area of the fovea the relation

between critical frequency and log I is a simple sigmoid curve whose middle section shows a slope of 11.0. Within the experimental error this slope is the same for light of all wave-lengths.

When the measurements are made with a peripheral retinal area containing rods and cones the relation between critical frequency and log I shows two clearly separable sections. Beginning with the lowest intensities, the relation is sigmoid and reaches an upper level or plateau at about 10 cycles per second which remains constant for 1.25 log units. This is followed by a second sigmoid relationship comparable to that given by the rod-free area alone. These two parts of the data are clearly separate functions of the rods at low intensities and of the cones at higher intensities.

This conclusion is confirmed in two ways. First the separation of the two sections of the relationship depends on the retinal location. The farther out in the periphery the measuring area is, the greater the separation of the two sections. Thus the greater the ratio of rods to cones, the farther apart are the two sections. Second, the separation of the two sections depends on the wave-length. For violet light they are very far apart, and approach each other as the wave-length becomes longer. The rod-portion disappears almost entirely with red light even though the retinal area used contains a large number of rods.

The present paper will appear in full in The Journal of General Physiology.

* Donders Fellow.

16. Color Vision in the House Fly (Musca Domestica). ZARMAIR ZAKARIAN, *343 East 24th St., New York, N. Y.*

It has been reported[1] that colored glasses are deterrent to the house fly, red and yellow being the best deterrents, and blue and green not nearly so effective. Much of the effect, however, can be ascribed to temperature.

The reaction of flies to colors is not quite unknown in the cleaning and dyeing industry, where one has often to deal with fly specks deposited upon fancy garments. This report is the result of a prolonged and more or less systematic study of the problem, the experiments having been made under *actual working conditions* as we find them in cleaning and dyeing establishments. The dresses were mostly silk and artificial silk—viscose rayon, and cellulose acetate rayon,—and some cotton, both grosgrain (dull) and satin (luminous) for each color. The reactions of the flies were noted in the dry as well as in the wet condition, that is, by spraying a small area upon the dresses with water, and, also, with a solution of alcohol in water; and the data were collected by counting the fly-specks deposited in equal times upon garments of various colors, in reflected as well as in diffused light. The results are as follows: Flies respond to white color and all the lightest tints, white and cream being *equally* preferred. Their response to the wet is immediate, and they always prefer the wet area to the dry on the same dress, even when plain water is used in the spray; and satins to crepe, if they are the same color, showing that the reactions of the insects are governed by *relative luminosity.* In reflected as well as diffused light, the reactions to different colors are in the same order. Color and luminosity being the same, flies prefer silk, cotton, etc., to *cellanese* in the ratio of 117 : 1, definitely establishing that their reactions are governed by *relative warmth,* even when there exists *no* temperature difference between two or more equally luminous garments of exactly identical color. From these and other facts, the conclusion, among others, is drawn that the reaction to color and luminosity in the fly is *subordinated* to its reaction to temperature, this latter sense governing its movements toward light. The relativity of both these senses seems to indicate that Weber's law is applicable in this case; that, there is no hue perception in the fly, and that its color vision consists of just the gray sensation in all gradations between black and white. It is worthy of note that flies do not seem to differ as widely, if any, in their sensitivity to variations in luminosity as humans do in their visual sensitivity to each of the three primaries.

[1] Nature, p. 277, Aug. 15, 1931. Also, Pilkington Brothers Ltd., *ibid.*, p. 529, April 5, 1930. Buchanan Smith, *ibid.*, p. 780, May 24, 1930. Beckett, *ibid.*, p. 780, May 24, 1930.

17. Microscopic Study of Pigments with Ultraviolet Light. GEORGE S. HASLAM AND CLYDE H. HALL, *Research Division, The New Jersey Zinc Company.*

The idea of using the ultraviolet microscope in order to study the component parts of two pigment systems was suggested by Green some years ago. The development of the technique to the necessary degree of refinement has been slow, but seems now to have reached a point where valuable information is obtainable.

This article shows photomicrographs of lithopone, titanium-barium, titanium-calcium pigments in which the ZnS or TiO_2 is pictured as opaque particles and the barium or calcium sulfate as transparent particles. This permits the investigator to study the relative size of each component as well as the amount of coalescence.

The present paper will appear in full in The Journal of the Optical Society of America.

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